Homework for
Mathematics 139.01
Advanced Calculus I
Fall Semester 2007

Homework No. 13
This assignment will not be graded.

Reading:
  Chapter 6, Section 6.4.

Recommended problems:
**Homework No. 12**  
Due Tuesday, December 4.

Reading:  
- Chapter 6, Section 6.2;  
- Chapter 6, Section 6.3.

Problems:  
- Section 6.2, pages 236-237: 1, 9, 10, 13, 16a;  
- Section 6.3, pages 242-244: 2, 3, 5, 11.

*Please remember to affirm the Duke Community Standard.*

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**Homework No. 11**  
Due Tuesday, November 27

Reading:  
- Chapter 6, Section 6.1.

Problems:  
- Section 6.1, pages 227-228: 1(a-d), 2, 7, 9, 10

Supplementary problems:  
1. Let $||f||_p$ be the $L^p$ norm on $[0,1]$. Suppose that $f(x) = x$. Show that $\lim_{p \to \infty} ||f||_p = ||f||_\infty$.

2. Prove that the text’s definitions of $\lim sup a_n$ and $\lim inf a_n$ are equivalent to the alternative definitions that were provided in lecture on November 20. Part (a): problem 9 on 228 (already assigned above, and partly done in class on November 20). Part (b): Make provision for the cases of unbounded sequences.

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**Homework No. 10**  
Due Tuesday, November 20

Reading:  
- Chapter 5, Sections 5.1, 5.2, and 5.3.

Problems:  
- Section 5.1, pages 168-169: 1, 2, 7, 12.  
- Section 5.2, pages 173-175: 1, 2(a,b), 6, 11.  
- Section 5.3, page 181: 1, 2, 3, 4.

Supplementary problem:  
Show that if $f(x, y) = e^{xy}$, then Equation (8) in Theorem 5.2.4 (page 172) holds.

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**Quiz 2 will be given on Thursday, November 1.**  
The quiz question or questions will be from Sections 3.2, 3.3, 3.5, or 3.6.

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**Homework No. 9**  
Due Tuesday, November 6
Reading:
Chapter 4, Sections 4.3 and 4.5.
Problems
Section 4.3, pages 139-140: 1, 4, 5, 7, 8, 9, 10, 11.
Section 4.5, pages 150-151: 1, 6, 8.

Test 2 will be given on Thursday, November 8.
This test will cover Sections 3.2, 3.3, 3.5, 3.6, 4.1, and 4.2.

Homework No. 8
Due Tuesday, October 30
Reading:
Chapter 4, Sections 4.1 and 4.2.
Problems:
Sections 4.1 and 4.2.
Pages 127-128: 2, 4, 7, 9, 13.
Pages 133-134: 5, 7, 10, 12, 13.

Supplementary Problems:
For Section 4.1.
Define \( f \) on the real numbers as follows: let \( f(x) = 0 \) if \( x \) is a rational number; let \( f(x) = x^2 \) if \( x \) is not a rational number.
(A) Find the subset of the real numbers (if any) where \( f \) is continuous. Use the definition of continuity to prove your finding.
(B) Find the subset of the real numbers (if any) where \( f \) is differentiable.
Use the definition of differentiability to prove your finding.
(C) Explain your results in terms of Theorem 4.1.1, and also in terms of the contrapositive of that theorem.

For Section 4.2.
Define \( f \) on the real numbers as follows: let \( f(x) = 1 \) for \( x \in (-\infty, 1) \); let \( f(x) = x^3 \) for \( x \in [1, +\infty) \).
(A) Find the largest set on which \( f \) is continuous and prove your result.
(B) Find the largest set on which \( f \) is \( C^1 \) (i.e., continuously differentiable) and prove your result.
(C) Find the largest set on which \( f \) is \( C^\infty \) (i.e., has continuous derivatives of all orders), and prove your result. What are the derivatives?
(D) Find the \( C^1 \) function \( F \) defined on the real numbers so that \( F' = f \) on the real numbers and so that \( F(1) = 0 \). Write the function \( F \) explicitly and sketch its graph.
(E) Find the function \( F_1 \) so that \( (F_1)' = f \) on the real numbers and so that \( F_1(1) = 1 \). Write the function \( F_1 \) explicitly and sketch its graph.
(F) Find the \( C^2 \) function \( G \) defined on the real numbers so that \( G'' = f \) on the real numbers and so that \( G(1) = \pi \). Write the function \( G \) explicitly and sketch its graph.

Homework No. 7
Due Tuesday, October 23
Reading:
Chapter 3, Section 3.5.
Problems:
Sections 3.5.
Pages 111-112: 2, 3, 4, 6, 9.
Supplementary problem:
Suppose that \( f(x) \) is defined on \([0, 1]\) as follows: For \( n = 0, 1, 2, 3, 4, \ldots \), \( f(x) = 2 - (1/2)^n \) if \( x \in S_n \), where \( S_n = [1 - (1/2)^n, 1 - (1/2)^{(n+1)}] \). Note that \( S_n \subset [0, 1] \) for each \( n \). For \( x = 1 \), define \( f(1) = 2 \).
(A) Sketch the graph of \( f \).
(B) Prove that \( f \) is not continuous on the interval \([0, 1]\), by showing that it fails to be continuous at one point in \([0, 1]\).
(C) How many points of discontinuity does \( f \) have on \([0, 1]\)?
(D) Is \( f \) Riemann integrable?
(E) Explain your answer to (D) and in terms of upper and lower sums.

Reading:
Chapter 3, Section 3.6.
Problems:
Sections 3.6.
Pages 117-118: 1, 3, 6.
Project paper proposal ("Report No. 4") due Thursday, October 18.
In this report you are to make a proposal, for my approval, for the topic of your project paper. The project paper can be about a mathematician who made a major contribution to calculus, about the history of a concept central to calculus, about an aspect of calculus that is beyond the scope of this offering of Math 139, or about an application of calculus in pure or applied mathematics. The project paper must be typewritten, double-spaced, not less than five pages in length (and not more than ten pages), excepting the title page, figures, and references. The submission of a project paper is a requirement for passing the course. The due date for the project paper is yet to be determined, but will be no earlier than about the middle of November.

The proposal for your project paper should be double-spaced and about (and no more than!) two typewritten pages; the two pages should include the references. Use one-inch margins and standard-size type (10 to 12 points); references can be in smaller type. Submit in hard copy only; print double-sided if you can. You may use web-based references, but exercise caution: I urge you to use trustworthy reference works and books. Three or more references are recommended.

Homework No. 6
Due Tuesday, October 16
Reading:
Chapter 3, Section 3.3.
Problems:
Pages 93-95: 1, 2, 3, 4, 6, 7, 9, 12, 13, 15

Homework No. 5
Due Thursday, October 11
Reading:
Chapter 3, Section 3.2.
Problems:
Page 86: 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12.

Brief report No. 3, due Thursday, October 4
Sir Isaac Newton and Gottfried Leibniz are generally credited with having developed (or, invented) "the calculus." Write a brief report in which you summarize the specific contributions to the calculus made by each.

The report should be double-spaced and about (and no more than!) two typewritten pages; the two pages should include the references. Use one-inch margins and standard-size type (10 to 12 points); references can be in smaller type. Submit in hard copy only; print double-sided if you can. You may use web-based references, but exercise caution: I urge you to use trustworthy reference works and books. Three or more references are recommended.

Please remember to affirm the Duke Community Standard.

Test 1 will be given on Tuesday, October 2.
This test will cover Chapter 1, Chapter 2 (except for sections 2.3 and 2.7), and Section 3.1 in Chapter 3.

Homework No. 4
Due Tuesday, September 25
Reading:
Chapter 2, Section 2.6;
Chapter 3, Section 3.1.
Problems:
Page 59: 1, 2, 4, 8
Pages 79-80: 2, 4, 5, 6, 8, 11, 13
Supplementary problem:
Suppose that the domain of a function $f$ is all the real numbers (thus, $Dom(f) = \mathbb{R}$). Further, suppose that $f(x) = x$ for all rational numbers $x$, and $f(x) = -x$ for all irrational numbers $x$.

(a) On what subset (if any) of $Dom(f)$ is $f$ continuous? Prove your answer.
(b) On what subset (if any) of $Dom(f)$ is $f$ not continuous? Prove your answer.

Please remember to affirm the Duke Community Standard.

Brief report No. 2, due Thursday, September 20
Prepare a brief report about Zeno's paradoxes of motion (or, perhaps, one of these paradoxes). Explain how the paradoxes (or paradox) have been resolved within the context of modern mathematics and/or have influenced the development of mathematics. Note that there may be differing perspectives on how to resolve the paradoxes, or on whether the paradoxes have been (or can be) satisfactorily resolved by modern mathematics.

The report should be double-spaced and about (and no more than!) two typewritten pages, including references. Use one-inch margins and standard-size type (10 to 12 points); references can be in smaller type. Submit in hard copy only; print double-sided if you can. You may use web-based references, but exercise caution: I urge you to use trustworthy reference works and books. Three references sounds about right to me. Please remember to affirm the Duke Community Standard.

Homework No. 3
Due Tuesday, September 18
Reading:
Chapter 2, Sections 2.4 and 2.5
Problems:
Pages 50-51: 1, 2, 5, 13
Pages 54-55: 1, 2, 3, 4

Homework No. 2, due Tuesday, September 11
Reading: Chapter 2, Sections 2.1 and 2.2
Problems:
Pages 33-34: 1, 2(b,d), 3(a,b), 4(a,d), 5, 8
(For problem 4, don’t use theorem 2.25 or 2.2.6; instead, use the definition of convergence to demonstrate the result.)
Page 39: 4, 6, 8
(Surely I expect you to be able to do problems like 1 and 2, but they are not assigned.)

Brief report No. 1, due Thursday, September 6
Prepare a brief report about any aspect of the transcendental numbers. The report should be double-spaced and about (and no more than) two typewritten pages, including references. Use one-inch margins and standard-size type (10 to 12 points). Submit in hard copy only; print double-sided if you can. You may use web-based references, but exercise caution. I urge you to use trustworthy reference works and books; this is an opportunity to explore the university’s mathematics collection. Three references sounds about right to me.

Homework No. 1, due Tuesday, September 4
Reading: Chapter 1
Problems:
Page 5: 10 and 11
Page 19: 2 (find a function that establishes a 1-1 correspondence) and 3
Page 20: 6 and 7
Page 24: 4 and 7
Page 25: 9 and 10
Reminders

Homework policies
Reading assignments and homework problems will be assigned weekly (approximately); homework problems will normally be due on Tuesdays. Homework should be written in pencil or black ink, and it must be submitted in hard copy (typewritten homework will, of course, also be accepted). All pages must be firmly fastened together.

Homework must be legible.

Problem solutions must be written in complete sentences.

Students are strongly urged to work in groups!
(However, copying of another student’s work will be considered to be a violation of the Duke Community Standard.)

Late Homework will not be accepted.
Exceptions: a student who cannot submit work on time due to a short-term illness must notify the instructor promptly by following the Short-Term Illness Notification Policy. In the case of a long-term absence, a student must present a dean’s excuse, in accordance with standard university policy. Details are in the Duke University Bulletin of Undergraduate Instruction.

In addition to homework problem sets, some brief report papers may be assigned. These must be type-written and double-spaced, and they will be subject to the same late policy as described (above) for problem sets.

Honor and Character
On all submitted work students are required to indicate compliance with the Duke Community Standard by writing “DCS” beside their signature.

It is the policy of Duke University, and the belief of the instructor, that honesty and the cultivation of good character are more important than academic achievement or the appearance of academic achievement.