Mathematics 287
Fall 1998
Problem Set 3

1. Show that if τ, σ are stopping times with respect to the filtration $\{\mathcal{F}_n\}$, then so are

$$\tau \wedge \sigma = \min\{\tau, \sigma\},$$

and

$$\tau \vee \sigma = \max\{\tau, \sigma\}.$$

2. Suppose M_n is a martingale with respect to the filtration $\{\mathcal{F}_n\}$. Suppose $\mathbf{E}(M_n^2) < \infty$ for each n. Show that

$$\mathbf{E}[(M_n - M_0)^2] = \sum_{j=1}^{n} \mathbf{E}[(M_j - M_{j-1})^2].$$

(Hint: you may first want to show for every $j < k$,

$$\mathbf{E}[(M_j - M_{j-1})(M_k - M_{k-1})] = 0. \quad)$$