

Mathematics 149S
Fall 1997
Problem Set 8

1. Evaluate

$$\int_0^{\infty} \frac{e^{-x} \sin x}{x} dx.$$

2. Find

$$\lim_{x \rightarrow \infty} x \int_0^x e^{t^2 - x^2} dt.$$

3. Do there exist positive integers a, b such that

$$a^2 + 3ab - 2b^2 = 122 ?$$

4. For which values of a, b, c, d is it the case that the polynomial

$$x^6 - 6x^5 + ax^4 + bx^3 + cx^2 + dx + 1,$$

has only positive roots?

5. Which is larger $60^{1/3}$ or $2 + 7^{1/3}$? (No calculators allowed!!!!)

6. Canadian Mathematical Olympiad (1988). Let $\{a_1, \dots, a_r\}$ denote a sequence of integers. For each non-empty subsequence A of S , we define $p(A)$ to be the product of all integers in A . Let $m(S)$ be the arithmetic average of $p(A)$ over all non-empty subsets A of S . If $m(S) = 13$ and $m(S \cup \{a_{r+1}\}) = 49$ for some positive integer a_{r+1} , determine the values of a_1, \dots, a_r and a_{r+1} .