Mathematics 149S
Fall 1997
Problem Set 7

1. Show that if n is an integer greater than 1, then n does not divide $2^n - 1$.

2. Show that there exists a number n such that the decimal representation of 2^n contains at least 1993 consecutive 0's.

3. Find the smallest integer n such that $2^n - 1$ is a multiple of 47.

4. Determine all polynomials $P(x)$ such that $P(x^2+1) = (P(x))^2 + 1$ and $P(0) = 0$.

5. Let $a, b, p_1, p_2, \ldots, p_n$ be real numbers with $a \neq b$. Define $f(x) = (p_1 - x)(p_2 - x) \cdots (p_n - x)$. Show that

$$\det \begin{bmatrix} p_1 & a & a & a & \cdots & a & a \\ b & p_2 & a & a & \cdots & a & a \\ b & b & p_3 & a & \cdots & a & a \\ b & b & b & p_4 & \cdots & a & a \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ b & b & b & b & \cdots & p_{n-1} & a \\ b & b & b & b & \cdots & b & p_n \end{bmatrix} = \frac{bf(a) - af(b)}{b-a}.$$