Mathematics 149S
Fall 1997
Problem Set 6

1. Suppose three circles (not necessarily of the same size) cover a square of side one. What is the minimum total area of the three circles?

2. Suppose a regular 1997-gon is drawn with center $(0,0)$ and vertices $A_1, A_2, \ldots, A_{1997}$ with $A_1 = (1,0)$. Let $B = (1997,0)$. Let d_i be the length of the line segment joining A_i and B. Find

$$d_1 d_2 \cdots d_{1997}.$$

3. Find the area of a convex octagon that is inscribed in a circle such that four consecutive sides have length 3 and the other four sides have length 2 .

4. Let b and c be fixed real numbers, and let the ten points $(j, y_j), j = 1, 2, \ldots, 10$, lie on the parabola $y = x^2 + bx + c$. For $j = 1, 2, \ldots, 9$, let I_j be the point of intersection of the tangents to the given parabola at (j, y_j) and $(j+1, y_{j+1})$. Determine the polynomial function $y = g(x)$ of least degree whose graph passes through all nine points I_j.