Mathematics 149S
Fall 1997
Problem Set 5
(In Class Portion).

1. Find

$$\sum_{j\equiv 0 (\mathrm{mod}\ 3)} \binom{n}{j}.$$

2. Let $f(x)$ be differentiable on $[0,1]$ with $f(0) = 0$ and $f(1) = 1$. For each positive integer n show that there exist distinct points $x_1, \ldots, x_n$ in $[0,1]$ with

$$\sum_{j=1}^{n} \frac{1}{f'(x_j)} = n.$$

3. (A question that arose in considering a gift exchange that occurs in the math department on Groundhog Day) The following game is played with 100 players. Numbers 1 through 100 are put into hat and each person chooses one number (without replacement). Players look at their number but do not see anyone else's pick. After looking at their numbers, players have the option of either: keeping the number forever, in which case they get a payoff in dollars of equal to the number, or putting the number back into the hat. (All players must make their decision simultaneously). After the numbers are put back in the hat all players still playing choose a number from this collection. Again, players can keep their number or put in back in the hat. The game ends when only one player still wants to put the number in the hat.

— Assuming everyone plays optimally, what is the best strategy for a given player?

— (this is not really a mathematical question but is a practical one) Assuming that the players are randomly chosen from the population and have only been told the rules immediately before the event, what is the best strategy?