Mathematics 149S
Fall 1997
Problem Set 5
Two Hours

This is a practice test. You are allowed two hours for these six problems.

1. Suppose $F(x)$ is a polynomial with integer coefficients such that none of the integers $F(1), \dots, F(k)$ is divisible by k. Show that for every integer n,

$$F(n) \neq 0.$$

2. Find

$$\lim_{n \to \infty} \int_0^1 \sqrt{1 + (n+1)^2 x^{2n}}\, dx.$$

3. Let $ABCDEF$ be a hexagon in a circle of radius r. Show that if $AB = CD = EF = r$, then the midpoints of BC, DE, and FA are the vertices of an equilateral triangle.

4, Does there exist a continuous function $f : [0,1] \to [0,1]$ such that for each $y \in [0,1]$ the set

$$\{x \in [0,1] : f(x) = y\}$$

is infinite?

5. Suppose f is a twice continuously differentiable function defined for all real numbers such that $|f(x)| \leq 1$ for all x and

$$(f(0))^2 + (f'(0))^2 = 4.$$

Show that for some real number x_0

$$f(x_0) + f''(x_0) = 0.$$

6. Show that from every set of 200 integers you can choose a subset of 100 with the total sum of the subset divisible by 100.