Mathematics 149S
Fall 1997
Problem Set 2

1. Given the infinite sequence of integers $a_1, a_2, a_3, \ldots$ defined by $a_1 = 1, a_2 = 0, a_3 = -5$, and

$$a_n = 4a_{n-1} - 5a_{n-2} + 2a_{n-3}, \quad n \geq 3,$$

find a closed form expression for a_n.

2. A fair coin is tossed 1997 times. Find the probability that at no point during the tossing are two heads flipped consecutively.

3. True or false: There exists an infinite sequence of nonzero complex numbers $a_1, a_2, \ldots$ such that

$$|a_i - a_j| \geq 1, \quad i \neq j$$

and

$$\sum_{i=1}^{\infty} \frac{1}{|a_i|^3} = \infty.$$

4. Prove that there exist integers a, b, c not all zero and each of absolute value less than one million, such that

$$|a + b\sqrt{2} + c\sqrt{3}| < 10^{-11}.$$

5. Find the sum of the series

$$1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{6} - \frac{1}{8} + \frac{1}{3} - \frac{1}{10} - \frac{1}{12} - \frac{1}{14} - \frac{1}{16} + \frac{1}{5} - \cdots.$$

6. If l, m, n are positive integers, let

$$a(l,m,n) = \sum_{k=0}^{l} \binom{n}{k} (l+m-k)^{n-k} (k-l)^k.$$

Prove that

$$\sum_{l=1}^{n} a(l,m,n) = \frac{m+n+1}{2} a(n,m,n) - \frac{m+1}{2} m^n.$$