Teaching Statement

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Teaching Philosophy

Overview/Principles

The most effective teachers I have known are those whose classrooms are places of exploration, marked as much by their openness and willingness to work through their students' perspectives as by the clarity and simplicity of their expositions. *My approach to teaching is to illustrate the main points through supporting examples, and to lead students to these points through an ongoing conversation about our evolving course goals.*

Background

My experience sharing mathematics began at the PROMYS program for high school students interested in Number Theory, run by Prof. Glenn Stevens at Boston University. Through the six summers I spent there as a student and counselor, I learned how to explore mathematics independently (embodied in the philosophy "think deeply of simple things"), as well as how to understand and explain complex concepts using a few simple examples and by asking good questions. This ongoing mathematical dialogue forms the basis of my teaching style, and my interactions with students. More than anything else, I try to help students understand how to ask the right questions, and hopefully how to begin to answer these questions for themselves.

Lectures

In the classroom I try to maintain an informal atmosphere as much as possible, so that students feel they are in a safe environment and are comfortable asking even "stupid" questions (which don't exist!). I am usually proactive about learning students' names – and have several times asked students in larger classes if I can take their pictures to use as flash cards.

In each lecture I focus on conveying one or two main ideas, supported by motivating questions and simple examples. This illustrates the basic computational skills needed for problem solving, and provides a few carefully chosen examples to focus on for future discussions. Also, having a few examples/questions that appear in different forms throughout the course provides a valuable reference point and helps students to feel comfortable as the material becomes more technically challenging.

I also try to keep students engaged by regularly turning to them and asking down-to-earth questions like "Ok, but why do we care?" and "What is this good for?". These refreshing asides help to break the all-too-frequent monotony of math classes, and give students a sense of why the things they are learning are important. This approach is particularly valuable when trying to teach students how to solve problems *whose solution is not immediately obvious.* Whenever we do problems together, I always start with the two questions: "What do we know?" and "What do we want?". By carefully answering these questions in the given context, students learn to think more independently and can develop strategies for solving previously unfamiliar problems using their existing knowledge.

Homework and Tests

Because I hope to teach students both the how to compute examples and how to use the ideas to approach new examples, my tests are usually fairly balanced with "doing-type" problems and "thinking-type" problems. Though difficult at first, students find problems where they are asked to "think outside the box" very rewarding in the long run, because it helps them to understand more deeply what it is they know, and how it can be applied to new situations.

To help students prepare for the exams, I have made an extensive series of review sheets which illustrate all of the ideas needed to do very well on the tests (though the problems may be different). At students's request I often also hold late-night review sessions which go over how to do these review problems, and typically last about 3 hours. These together with homework problems (whose solutions are posted on the course website) provide a rich source of examples for students to test their skills and understandings, and form the basis of the exams. Finally, when returning graded exams I also distribute hand-written solutions which work through each question in a motivated step-by-step fashion.

Teaching Experience

In the past 7 years I have been fortunate to have the opportunity not only to teach a variety of undergraduate courses at three major universities, but also to design several of my own courses and improve existing ones. These experiences have been very valuable for me personally, and have shown this teaching style to be highly effective at motivating students to engage course material through their natural curiosity to understand what is going on. Rather than describe each course in detail, below I will describe some of my most rewarding and unique experiences.

Cryptography Seminar(Rutgers/Duke)

This course was designed as a more mathematical replacement for an earlier discussion-based "math for poets" course at Rutgers discussing cryptography and society. I later proposed and taught this as a freshman seminar at Duke.

The first half of the course focused on classical cryptography schemes (letter relabeling and rearrangement) and various statistical attacks for breaking them. Students were responsible for evaluating the security of these schemes, and for cracking them if they were insecure. These ciphers included the shift cipher, cryptogram (without spaces), permutation cipher, Vigenère cipher, and the one-time pad. The second half of the course focused on public-key cryptography and developing the necessary proficiency with modular arithmetic needed to implement these. Mathematically, this included a discussion of primes, units, multiplicative inverses, the theorems of Fermat and Euler, and primitive roots. With this, we covered RSA, Diffie-Hellman secret sharing, and ElGamal. The students were responsible for creation of their own public/private keys, encryption and decryption, digital signatures, and key management issues. We also discuss related security issues such as: what is a "hard" problem, why ciphers are based on "hard" problems, and the man-in-the-middle attack.

All computations were essentially done by hand (students were allowed to use a simple calculator, but no computer programs), which gave a tangible appreciation for the computational complexity of various cryptographic operations and ensured that students really knew what they were doing. It was impossible to find an adequate text for the material, so there were many handouts. To help with this, I created a course web page with homeworks and many lecture summaries as well as various C programs to compute the relevant statistics and aid in the creation of the exams. While primarily designed to help students, these will be a valuable resource for the next instructor and will provide a sense of continuity and a solid starting point for any future changes.

This more rigorous style in a course for non-math majors comes with the possible pitfalls of discouraging those with weaker backgrounds and obscuring the issues with mindnumbing mathematics. However students seemed quite interested in this more hands-on approach, and the course has consistently received high evaluations, with comments like: "Jon allowed me to look at math like I have never seen it before", "The teacher was outstanding", and "If I were stranded on a desert island, I would want him with me".

Calculus I and Linear Algebra Course Coordinator (Princeton)

Over two semesters, I was responsible for improving and coordinating the Calculus I and Linear Algebra courses at Princeton University. In addition to revising the syllabus and homeworks, I made a comprehensive effort to improve the consistency of the course (across all 10 sections) and to provide more review resources for students than the usual "classroom/office hours" model provides. This involved working with the Director of Undergraduate Studies to setup weekly lunch meetings for instructors to share ideas about the material for the upcoming week, and arrange for all of the lectures (from one section) to be videotaped and made available to students for viewing online. I also created more review opportunities for students in the form of weekly review sessions, a comprehensive set of review problems for the exams with online answers, and several days of 2-3 hour review sessions before exams both for course review and problem solving. While not requiring substantially more man-power to implement, these changes greatly improved the ability of the course to accomodate students' varied study habits, which was particularly important given its predominantly freshman enrollment. These changes were well-received by students, and will hopefully become a lasting model for the way these courses are taught in future years.

Number Theory Seminar (Duke)

At Duke I had my first opportunity to teach an undergraduate course in Number Theory for 3 semesters (and again in Spring '06). This course was modeled on my experiences at the PROMYS program, with the idea of giving students as much of an idea of exploration and independent discovery as possible. The course was a hands-on introduction to some important concepts and proofs in classical number theory, focusing on primes and prime factorization in $\mathbb{Z}$ and $\mathbb{Z}[\sqrt{-d}]$, quadratic residues and reciprocity, and representing numbers in the form $x^2 + dy^2$. Students were also responsible for writing a 10-15 page paper describing/proving some major result in number theory and giving an hour-long presentation about it to their peers. To help with this, each student was given an initial project outline with references, and had weekly half-hour meetings with me to discuss their progress and ask questions. Students found this research-type experience very valuable, particularly because it allowed them to understand something over time that at first seemed completely unintelligible, making comments like: "Thanks for a great class. The best I've ever taken!" and "I honestly believe that classes like yours have inspired me to continue to get my Masters and I really do appreciate the enthusiasm you shared with me during my days at Duke."

Independent Study Projects (Duke)

In addition to the many independent-style final projects for the Number Theory class, I have had two students take independent study courses exploring some aspect of number theory for themselves.

The first student (Mayank Varia) was interested in learning about modular forms after doing a final project on the arithmetic of elliptic curves for my number theory class as a junior. Over the following 2 semesters he learned the basic theory of modular forms for congruence subgroups of $\mathrm{SL}_2(\mathbb{Z})$ including the moduli and double coset interpretation of Hecke operators, the theory of newforms, congruence zeta functions and the Weil conjectures, how to resolve singularities via blowups, and some basic facts about l-adic Galois represention. This exploration culminated in a senior thesis on "The explicit computation of the L-function of a Kummer surface", which identifies a modular form whose symmetric square L-function essentially gives the L-function attached to (the middle cohomology of) a certain Kummer surface E, and he shows numerically that these agree at many fourier coefficients by counting points on E over the finite fields $\mathbb{F}_{p^r}$ for some small primes p.

The second student (Mandy Frese) was a sophomore interested in learning about cryptography after some informal discussions during my Calculus III class. During our semester-long independent study she learned basic properties of modular arithmetic through quadratic reciprocity, encrypted messages using several popular public-key protocols by hand (RSA, Diffie-Hellman Secret sharing, and El Gamal), and explored several sieving methods used for factoring. The following semester she took my number theory class where she learned the basics of the arithmetic of elliptic curves over finite fields and did a final project on elliptic curve cryptography. Her project focused on explicit computations with certain curves over $\mathbb{F}_p$ and a comparison with the usual discrete log problem over $\mathbb{F}_p^\times$. She then continued with another independent study where she studied (and partially implemented) a cryptographic hash function of K. Lauter based on traversing the graph of (p-isogenous) of supersingular elliptic curves for some large prime p.

Graduate Student Support (Rutgers/Princeton/Duke)

In terms of graduate education, through the VIGRE program at Rutgers I led a semester-long reading course with two graduate students at Rutgers interested in number theory. They read most of Gouvea's book "p-adic numbers", and then gave a 30 minute presentation on a related topic. One was an introduction to p-adic numbers, and the other was an application of p-adic techniques to the Hasse principle for quadratic forms over $\mathbb{Q}$. I have also tried to encourage and assist graduate students in learning new topics. Along these lines, I led an informal weekly support seminar for graduate students who were having their first experiences in algebraic geometry, which lasted for one semester. This involved discussing problems and ideas from the first two chapters of Hartshorne's book, with a focus on the classical phenomena that motivate the more sheaf-theoretic language of the subject. Additionally, I have been closely involved with several informal graduate student seminars focusing on various aspects of automorphic forms. At Duke there are few graduate students in number theory, so in Fall 2005 I gave a series of general interest lectures at the recently revived Graduate/Faculty Seminar on "The p-adic way of life", describing the arithmetic of p-adic numbers, the basics of p-adic analysis, and how they can be used to better understand the integers. In Fall 2006, this was followed by a talk on "How many ways can you write a number as a sum of 4 squares?", which used p-adic/local methods to find the number of ways of writing a 1 and 2 as a sum of 4 squares.