Teaching Statement

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1 Introduction

I love mathematics and I love teaching it. I love it when students demonstrate to me that they understand the structure behind an argument. I love it when students come up with different techniques for solving problems. I love it when students ask questions. I love it when students *think*.

2 Problem-Solving

I believe that one of the key components of mathematics is problem-solving. It is important for students to understand mathematical theory and to be able to do calculations, but not nearly as important as to acquire critical thinking skills. Students will forget formulas soon after the course is over, but problem-solving skills can aid them for the rest of their lives.

Teaching students how to solve problems can be a daunting task. Experience has taught me that pure lecture is not the best method for getting students to think critically. The only way for students to become better problem solvers is by solving problems. To give students practice thinking, I sometimes have them work together on hard problems in class. I answer questions and help groups get past the sticky points. Students are forced to communicate about mathematics, and to do so they must *think*. Such an exercise is valuable even if students don't obtain the answer. At the very least, students have a better knowledge of what they do and don't understand. The solution to the problem means more to students when they have been thinking about it.

Group work has other benefits. Group work gives students a break from listening to lecture and helps them to stay focused. A combination of lecture and group work accommodates a broader range of learning styles. Group work fosters a more relaxed environment and makes it easier for students to ask questions. Students benefit from having other students explain things to them. Also, students greatly benefit from explaining concepts to others. Working as a team is itself a valuable skill that will apply to other courses in college and even beyond.

In addition to group work, I encourage critical thinking through a series of homework assignments called "Fun Exercises." Let me give an example of one such assignment. A typical Calculus II class covers many different tests for series convergence, and it can be difficult for a student to know which test to try on a particular series. One solution is to give students a comprehensive set of guidelines detailing which tests work well for different types of series. Such a teacher-generated set of guidelines squanders an opportunity for students to *think* for themselves. Instead, I ask for students to give me an algorithm for testing series for convergence or divergence. I make it clear that there are many possible approaches to

this project, like constructing bulleted lists or designing complex flowcharts. I challenge students to create guides that apply to every series that they are likely to encounter. This "Fun Exercise" molds itself to each individual student's strengths. Students get practice thinking critically and gain a better understanding of all the different convergence tests. I believe that students benefit more from thinking about the patterns on their own than from memorizing a list given to them.

3 Helping Students Connect with Mathematics

I find the ideas in mathematics beautiful and compelling enough to motivate me to learn them, but the same is not necessarily true for my students. Some students need applications to motivate them to learn mathematics.

To help these students connect with mathematics, I draw on examples from chemistry, economics, and physics, to name a few. Students are more motivated to learn the material if it relates to one of their interests. One student wrote in a midterm evaluation, "Math doesn't mean a whole lot to me (and I don't do well in it) if it is not placed in a real world context. So I like how the fun exercises bring math out of a problem set." In addition, most students in lower-level mathematics courses will major in something other than mathematics. It is helpful for these students to understand how their respective disciplines use mathematics.

It is virtually impossible to find an example that will resonate with every student. I have developed one very successful "Fun Exercise" to overcome this obstacle. I introduce a topic with many applications, like basic probability or definite integration. I ask each student to design an example problem and solution on the topic. I encourage students to be creative and to choose subjects that they are passionate about. When students are interested in a project, they are more likely to spend a lot of time and energy on it. I have seen not one but *two* examples inspired from Tolkien's *Lord of the Rings* trilogy!

This exercise is a pleasure to grade because the students' enthusiasm is evident. After grading it, I know about one interest of each of them. It engages students with skills different than those typically valued in mathematics classes. Furthermore, students must understand the topic very well in order to create examples. The assignment requires higher-order thinking and is fun for me and for my students.

4 Teaching with Technology

Technology brings exciting new ways to teach mathematics. Calculators and computers help students digest complicated theory through the exploration of examples. Many students will need to use calculators and computers in their careers, and teachers should help acquaint them with the tools at their disposal.

The availability of graphing calculators has changed the way that Calculus is taught at many colleges and universities. I have taught three classes that require students to use

graphing calculators and one in which calculators were not allowed. Graphing calculators are helpful for exploring examples and developing intuition. For example, I often have students approximate the sum of a series by computing higher and higher partial sums. This reinforces the rigorous notion of convergence through basic examples. With the help of Dr. Jack Bookman, a faculty member serving with me on the Calculus committee, I turned a very successful calculator activity into a Calculus lab entitled "Approximating Definite Integrals." A copy is available at my website `http://www.math.duke.edu/~jaspive/` under the "Teaching" tab. In this lab, students calculate the error for various approximation techniques of definite integrals, like Left-Hand Sums and the Trapezoid Rule. Students discover that as the number of data points doubles for a Left-Hand Sum, the error is cut approximately in half. The formal error estimates for these techniques are introduced at the end of the lab. I believe that students understand and appreciate the theorems far better once they gain an intuitive grasp of their content. Students need concrete examples to augment the abstract theory, and technology can provide the means.

Students sometimes respond to technology in a way that they do not respond to other teaching methods. Let me recount an example from a course I taught on Linear Algebra and Differential Equations. The textbook did not adequately illustrate the applications of orthonormal bases. I related orthonormal bases to Fourier series, which many of the students had encountered in a prior course. I used Maple to graph a parabola along with higher and higher partial sums of the Fourier series to illustrate convergence. Students were amazed at how quickly the software computed the definite integrals for the Fourier coefficients and how it handled partial sums with a thousand or more terms. One student was so impressed by this example that he felt compelled to write the following in a Teacher Evaluation: "Spivey did an example that was sort of the derivation of Fourier series; it was very intriguing." Of all the examples that I used, this is the one that he remembered best. Technology-driven examples can win students' interest in a way that other examples cannot.

5 Conclusion

When I teach mathematics, my primary goal is to get students to *think*. To this end, I focus on making connections. Through group work and "Fun Exercises," students discover some of the beautiful connections inherent in mathematics. Together, we explore how mathematics is closely connected with real-world applications. Technology helps students connect concrete examples with abstract theory. Experience has taught me that these methods help students become better problem solvers. It brings me great joy when students *think*.