Richard Hain April 12, 2023

Math 612
Problem Set 6

Due: Tuesday, April 25, 2023

1. Suppose that M is a connected, closed manifold (not necessarily orientable). Show that if N is a connected closed submanifold of M of dimension d, then $M-N$ is connected when $d<n-1$ and that $M-N$ is connected if and only if $H_{n-1}(N;\mathbb{F}_2)\to H_{n-1}(M;\mathbb{F}_2)$ is injective.

2. Now suppose that M is oriented. Show that for each $d\in\mathbb{Z}$, $M-N$ is connected if and only if $H_{n-1}(N;\mathbb{Z}/d)\to H_{n-1}(M;\mathbb{Z}/d)$ is injective. Deduce that if the image of the fundamental class of N in $H_{n-1}(M;\mathbb{Z})$ is non-zero, it is primitive — that is, it is not divisible by any $d>1$.

3. Suppose that $n=p+q$, where n,p,q are positive integers. Is there an open subset of $\mathbb{R}^n$ that is homeomorphic to an open neighbourhood of (the standard embedding of) $S^p\vee S^q$ in $S^p\times S^q$?