

MATH 612
PROBLEM SET 5

Due: Tuesday, April 4, 2023

1. Suppose that M is a smooth manifold. Denote the space of smooth homotopy classes of smooth maps $f : M \rightarrow S^1$ by $[M, S^1]_\infty$. Fix a generator μ of $H^1(S^1; \mathbb{Z})$. Show that the map

$$[M, S^1]_\infty \rightarrow H^1(M; \mathbb{Z})$$

that takes f to $f^*\mu$ is a bijection.

Remark: It is true that if X is a CW-complex then the map

$$[X, S^1] \rightarrow H^1(X; \mathbb{Z})$$

is a bijection, where $[X, S^1]$ denotes homotopy classes of continuous maps.