BINOMIAL D-MODULES

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The authors dedicate this work to the memory of Karin Gatermann, friend and colleague

ABSTRACT. We study quotients of the Weyl algebra by left ideals whose generators consist of an arbitrary $\mathbb{Z}^d$-graded binomial ideal I in $\mathbb{C}[\partial_1, \ldots, \partial_n]$ along with Euler operators defined by the grading and a parameter $\beta \in \mathbb{C}^d$. We determine the parameters β for which these D-modules (i) are holonomic (equivalently, regular holonomic, when I is standard-graded); (ii) decompose as direct sums indexed by the primary components of I; and (iii) have holonomic rank greater than the rank for generic β. In each of these three cases, the parameters in question are precisely those outside of a certain explicitly described affine subspace arrangement in $\mathbb{C}^d$. In the special case of Horn hypergeometric D-modules, when I is a lattice basis ideal, we furthermore compute the generic holonomic rank combinatorially and write down a basis of solutions in terms of associated A-hypergeometric functions. This study relies fundamentally on the explicit lattice point description of the primary components of an arbitrary binomial ideal in characteristic zero, which we derive in our companion article [DMM08].

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