Math 602 Homework #2, Spring 2016

Instructor: Ezra Miller

Solutions by: ...your name...

Collaborators: ...list those with whom you worked on this assignment...

Due: 15 February 2016

Reading assignments

- by Thursday 4 February: §2.3, 2.4, 2.5 in [Eis95]
- by Tuesday 9 February: §3.1, 3.2, 3.3, 3.8 in [Eis95]
- by Thursday 11 February: the rest of Chapter 3 in [Eis95]
- by Tuesday 16 February: §4.1, 4.2, 4.3, 4.4 in [Eis95]

Exercises

An exercise whose label is of the form C.n refers to the n^{th} exericse in [Eis95, Chapter C].

2.4 (a)

(b)

(c)

2.11

2.13

2.17 (a)

(b)

(c)

(d)

2.19 (a)

(b)

2.21 (a)

(b)

3.3

3.5 (a)

(b)

(c)

3.6

3.12

Additional exercises.

1. Every irreducible monomial ideal is of the form $\mathfrak{m}^{\mathbf{b}} = \langle x_i^{b_i} \mid b_i \geq 1 \rangle$ for some vector $\mathbf{b} \in \mathbb{N}^n$, and these ideals are partially ordered by inclusion (see Exercise 3.6).

   (a) Given a monomial ideal I, show that there are only finitely many irreducible monomial ideals that are minimal among those containing I.

   (b) Prove that the intersection of these minimal irreducible monomial ideals equals I.

   (c) Conclude that I possesses a unique irredundant expression as an intersection of irreducible monomial ideals.

References

[Eis95] David Eisenbud, *Commutative algebra, with a view toward algebraic geometry*, Graduate Texts in Mathematics Vol. 150, Springer–Verlag, New York, 1995.