

# Math 602 Homework #1, Spring 2016

Instructor: Ezra Miller

Solutions by: ...your name...

Collaborators: ...list those with whom you worked on this assignment...

Due: 1 February 2016

An exercise whose label is of the form C. $n$  refers to the  $n^{\text{th}}$  exercise in [Eis95, Chapter C].

1.9

1.11 (a) Don't bother to do part (a), but read it and compare the figure with part (b).

(b)

1.18

1.24 (a)

(b)

(c)

(d)

1.25 (a)

(b)

(c)

2.3

15.2

15.4 (a)

(b)

15.29 (You may use Buchberger's algorithm by hand, or any computer algebra system you wish—this is the recommended method. If you use a computer, you must say which software you used, and which commands you used. The point is to begin learning to use the software.)

15.40

15.42

Additional exercises.

1. Find an algorithm to decide if a given ideal  $I \subseteq \mathbb{k}[x_1, \dots, x_n]$  contains a monomial.
2. Given a homomorphism  $\phi : F \rightarrow G$  of free modules (i.e., given a matrix of polynomials) and a submodule  $M \subseteq G$  (i.e., given a finite generating set consisting of vectors in  $G$ ), find an algorithm to compute a generating set for the preimage  $\phi^{-1}(M) \subseteq F$ . Your algorithm can be of the form, “construct a matrix from the given data, compute its kernel, and do some specified operation.”

## References

- [Eis95] David Eisenbud, *Commutative algebra, with a view toward algebraic geometry*, Graduate Texts in Mathematics Vol. 150, Springer–Verlag, New York, 1995.