Problem 1. The region $D$ in the figure is bounded by the curves

\[ xy = 1 \quad xy = 2 \quad \frac{y}{x} = 1 \quad \frac{y}{x} = 3 \]

Let $u = xy$ and let $v = y/x$.

(a) Show that $\frac{\partial(u, v)}{\partial(x, y)} = 2v$

(b) Compute $\text{area}(D)$

\[
\text{area}(D) =
\]
Problem 2. The region $R$ depicted in the figure is bounded by

$$x^2 + y^2 = 1 \quad x^2 + y^2 = 4 \quad y = x \quad y = -x$$

Let $\vec{F}$ be the vector field on $\mathbb{R}^2$ given by $\vec{F} = \begin{pmatrix} x + e^y \\ 3y + \cos(x) \end{pmatrix}$.

(a) Use a double integral to show that $\text{area}(R) = \frac{3\pi}{4}$

(b) Compute the flux $\Phi$ of $\vec{F}$ across $\partial R$.

\[ \Phi = \]
Problem 3. The figures above depict three curves $C_1$, $C_2$, and $C_3$. The curve $C_1$ is the circle of radius one centered at $(1, 0)$ and the curve $C_2$ is the line segment from $(-2, 1)$ to $(3, -1)$.

\(a\) Compute \(\oint_{C_1} \vec{E} \cdot d\vec{x}\) where $\vec{E} = \left( \begin{array}{c} y \cos(x) \\ \sin(x) + 3x \end{array} \right)$

\(b\) Compute \(\oint_{C_1} \vec{F} \cdot \vec{n} \, ds\) where $\vec{F} = \left( \begin{array}{c} 2y \cos(x) \\ y^2 \sin(x) + \pi \end{array} \right)$
(c) Compute \[ \int_{C_2} \vec{G} \cdot d\vec{x} \]
where \( g(x, y) = e^{x^2} + \sin(\pi y) \) and \( \vec{G} = \nabla g \)

\[ \int_{C_2} \vec{G} \cdot d\vec{x} = \]

(d) Compute \[ \int_{C_3} \vec{H} \cdot d\vec{x} \]
where \( \vec{H} = \left( \begin{array}{c} y^2 \\ 2xy + 6y^2 \end{array} \right) \)

\[ \int_{C_1} \vec{H} \cdot d\vec{x} = \]
Problem 4. Let $P$ be the region depicted in the figure. The centroid $(\bar{x}, \bar{y})$ of $P$ has coordinates

$$\bar{x} = \frac{25}{12}, \quad \bar{y} = \frac{5}{4}$$

(a) Show that area($P$) = 8

(b) Compute the volume $V$ of the solid obtained by rotating $P$ about the line $y = \frac{25}{4}$
Problem 5. The helicoid is the surface $S$ parameterized by

$$\vec{x}(u, v) = \langle u \cos v, u \sin v, v \rangle \quad 0 \leq u \leq 1 \quad 0 \leq v \leq 2\pi$$

The density at each point of the helicoid is measured by $\delta(x, y, z) = \sqrt{1 + x^2 + y^2}$.

(a) Show that $\vec{N} = \langle \sin v, -\cos v, u \rangle$ is normal to $S$ at every point $\vec{x}(u, v)$ of $S$.

(b) Compute the mass of $S$.

$$\text{mass}(S) = \quad$$
Problem 6. Let $T$ be the solid region in $\mathbb{R}^3$ bounded by

\[ x \geq 0 \quad y \geq 0 \quad z \geq 0 \quad x + 2y + z = 2 \quad z = 1 \]

The density at every point of $T$ is given by $\delta(x, y, z)$ and the mass of $T$ is $m$. Find, but do not evaluate, an iterated integral to compute the moment of inertia $I_y$ of $T$ around the $y$-axis.

\[ I_y = \]
**Problem 7.** Let $S$ be the surface in $\mathbb{R}^3$ consisting of the hemisphere $z = \sqrt{1 - x^2 - y^2}$ and the disk $x^2 + y^2 \leq 1$ in the $xy$-plane. Compute the flux $\Phi$ of

$$\vec{F} = \langle 4x^3z, 4y^3z, 3z^4 \rangle$$

through $S$. 

$$\Phi =$$
Problem 8. Let $C_1$ and $C_2$ be the curves defined by intersecting the cylinder $x^2 + y^2 = 1$ with the planes $z = 0$ and $z = 5$. Compute

$$W = \oint_{C_1} \vec{F} \cdot d\vec{x} + \oint_{C_2} \vec{F} \cdot d\vec{x}$$

where $\vec{F} = (2yz, e^y, xy)$.