Math 212

Multivariable Calculus

Sections 4 & 5

Exam I

Name:                                          Instructor:

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Score: ____
100
Problem 1 (12 Points). The figure to the right depicts the Lissajous curve

\[ \mathbf{r}(t) = \begin{pmatrix} \cos 3t \\ \sin 2t \end{pmatrix} \]

The indicated point is the position of \( \mathbf{r}(t) \) at time \( t = \pi \).

(a) Compute \( \mathbf{r}'(\pi) \) and \( \mathbf{r}''(\pi) \).

\[ \mathbf{r}'(\pi) = \quad \text{r''}(\pi) = \]

(b) Compute \( \cos \theta \) where \( \theta \) is the angle between \( \mathbf{r}'(\pi) \) and \( \mathbf{r}''(\pi) \).

\[ \cos \theta = \]

(c) Compute the curvature \( \kappa(\pi) \) of \( \mathbf{r}(t) \) at time \( t = \pi \).

\[ \kappa(\pi) = \]
**Problem 2** (12 Points). Each of the figures below depicts various level sets of a different function $\mathbb{R}^2 \rightarrow \mathbb{R}$.

For each of the following functions, circle the label of the figure that depicts its level sets.

**Function 1.** $g(x, y) = x^2 + 2y^2$

- (a) (b) (c) (d) (e) (f)

**Function 2.** $g(x, y) = x^2 - y^2$

- (a) (b) (c) (d) (e) (f)

**Function 3.** $g(x, y) = xy$

- (a) (b) (c) (d) (e) (f)

**Function 4.** $g(x, y) = 2x^2 + y^2$

- (a) (b) (c) (d) (e) (f)

**Function 5.** $g(x, y) = x - y^2$

- (a) (b) (c) (d) (e) (f)

**Function 6.** $g(x, y) = y - x^2$

- (a) (b) (c) (d) (e) (f)
Problem 3 (8 Points). Determine whether or not \( \lim_{(x,y) \to (0,0)} \frac{xy - y^2}{x^2 + y^2} \) exists.

Problem 4 (8 Points). Find the equation for the plane passing through \((3, 3, -1)\) which is orthogonal to the two planes \(x + y = 2\) and \(2x + z = 10\).
**Problem 5** (15 Points). Let \( S \) be the surface described by the equation

\[
x^2 - y^2 - z^2 = 2
\]

(a) Give a function \( f : \mathbb{R}^a \to \mathbb{R}^b \) whose level set \( f = 0 \) is \( S \).

\[
a = \quad b = \quad f =
\]

(b) Find an equation for the plane tangent to \( S \) at the point \((2,1,1)\).

(c) Show that \( x \) can be locally viewed as a function of \( y \) and \( z \) at every point of \( S \).

(d) Compute \( \frac{\partial x}{\partial y} \).

\[
\frac{\partial x}{\partial y} =
\]
Problem 6 (15 Points). An ant is crawling in the $xy$-plane with unit speed. The temperature at every point of the $xy$-plane is given by $T(x, y) = x^2 + 2y^2$. The ant starts at the point $P(3, 2)$.

(a) What rate of change in temperature will the ant experience if she chooses to travel in the direction of the vector $\langle -1, 1 \rangle$?

(b) In what direction should the ant travel to experience the greatest rate of increase in temperature?

(c) What is the greatest possible rate of increase in temperature that the ant could experience?

(d) Find a function $f : \mathbb{R}^a \to \mathbb{R}^b$ whose level set $f = 0$ is the graph of $T$. 

\[ a = \quad b = \quad f = \]
Problem 7 (10 Points). Suppose that $f(x, y)$ has continuous second-order partial derivatives with $x = e^r + s$ and $y = rs$.

(a) Compute $\frac{\partial f}{\partial r}$. Simplify your expression as much as possible.

(b) Compute $\frac{\partial^2 f}{\partial r^2}$. Simplify your expression as much as possible.
Problem 8 (10 Points). Let $V$ be the volume of the solid lying under the graph of $f(x,y) = y^2$ and above the region $D$ depicted in the figure.

(a) Write $V$ as an iterated integral.

(b) Compute $V$.
Problem 9 (10 Points). Let $f(x, y) = e^{x^2}$.

(a) Use the axes on the right to sketch the domain in $\mathbb{R}^2$ described by the bounds of the double integral $\int_0^1 \int_y^1 f(x, y) \, dx \, dy$.

(b) Find expressions for $a$, $b$, $c$, and $d$ so that $\int_0^1 \int_y^1 f(x, y) \, dx \, dy = \int_a^b \int_c^d f(x, y) \, dy \, dx$.

\[
a = \quad b = \quad c = \quad d = \quad
\]

(c) Compute $\int_0^1 \int_y^1 e^{x^2} \, dx \, dy$.

\[
\int_0^1 \int_y^1 e^{x^2} \, dx \, dy =
\]