Math 224: Scientific Computing

Homework 5: Eigenvalue Problems

Due: *Thursday, October 2, 2008*

1. Read Health 4.1, 4.2, 4.5.1–2

2. X-matrix, the sequel Let A be an $n \times n$ matrix (n must be an EVEN integer), whose entries are all zeros, except for

$$a_{i,i} = i, \quad a_{i,n-i+1} = i/2, \quad i = 1, 2, \ldots, n$$

   If you "`spy`" this matrix in `matlab`, you will see that the structure of this matrix is an "X." Let $n = 6$, but write all of your code so it could be generalized to any value.

   (a) Based on the $\|A\|_1$ and $\|A\|_\infty$ matrix norms, give estimates for what is known about the eigenvalues of this matrix.

   (b) Implement the Power method to find the largest eigenvalues of A, $\lambda_{\max} = \max|\lambda(A)|$. How many iterations does it take to converge to an answer?

   (c) Implement the Inverse Power method to find the smallest eigenvalues of A, $\lambda_{\min} = \max|\lambda(A)|$. How many iterations does it take to converge to an answer?
   Turn in print-outs of your code for (b) and (c).

   (d) The LAPACK routines for computing eigenvalues and eigenvectors for an $n \times n$ real non-symmetric matrix A are `dgeev.c` and `dgeev.f`. These subroutines can be downloaded from `www.netlib.org`. Use the appropriate subroutine to compute the eigenvalues of A.

3. Some comic relief: