Homework 8: Applications of numerical ODE IVP 3.0

Due: *Thursday, March 26, 2009*

1. Read Burden and Faires Chapters 5.5 and 5.9.

2. Systems of ODEs. The van der Pol equation prescribes the motion of a particle $x(t)$ by

$$x''(t) + \mu(1 - x(t)^2)x'(t) + x(t) = 0. \tag{1}$$

(a) Show that the second-order ODE (1) can be transformed into a 2×2 system of first-order ODEs

$$y_1'(t) = y_2(t) \tag{2}$$

$$y_2'(t) = \frac{1}{\epsilon}\Big(- y_1(t) + (1 - y_1(t)^2)y_2(t)\Big), \tag{3}$$

where $\epsilon = 1/\mu^2$.
(Hint: Make the transformation $y_1(t) = x(t)$, $y_2(t) = \mu x'(t)$ and $t = t/\mu$.)

(b) Integrate Eqs. (2) and(3) from $t = 0$ to $t = 0.5$, for $\epsilon = 10^{-1}, 10^{-3}, 10^{-5}$ using forward Euler and the following initial conditions:

ϵ	$y_1(0)$	$y_2(0)$
10^{-1}	2	-0.654321
10^{-3}	2	-0.66654321
10^{-5}	2	-0.6666654321

Find a time-step small enough so that your method is stable and that the error of your computed solution is $< 10^{-6}$ when compared with the following reference solution:

ϵ	$y_1(0.5)$	$y_2(0.5)$
10^{-1}	1.613344960228556	-0.9435973073267591
10^{-3}	1.59698077872837	-1.029103015777729
10^{-5}	1.596770525704802	-1.0303800156140487

What happen to the time step size as ϵ decreases.

(c) Repeat (b) using backward Euler. You will need a Newton-for-systems to solve the Jacobian equation. How small does your time step have to be for each ϵ to achieve stability and the same erorr tolerance?

(d) On three graphs (one for each ϵ), plot the solution (y_1, y_2) computed by backward Euler as a function of time.