Math 160S: Mathematical Numerical Analysis

Homework 7: Applications of numerical ODE IVP 2.0

Due: *Thursday, March 19, 2009*

1. RK-3. Derive the conditions on the unknown coefficients in the following 3-stage Runge-Kutta scheme needed to attain third-order accuracy for solving $dy/dt = f(t, y)$:

$$\begin{aligned} k_1 &= f(t_n, u_n) \\ k_2 &= f(t_n + \alpha_1 h, u_n + \beta_1 h k_1) \\ k_3 &= f(t_n + \alpha_2 h, u_n + \beta_2 h k_2) \\ u_{n+1} &= u_n + h(\gamma_1 k_1 + \gamma_2 k_2 + \gamma_3 k_3) \end{aligned}$$

2. Accuracy and Speed. Solve the nonlinear initial value problem for $y(t)$,

$$\frac{dy}{dt} = \sin(ty^2), \quad y(0) = 3$$

Do this four different ways:

- RK-1 (Forward Euler)
- RK-2 (your choice)
- RK-3 (your choice of favorite coefficient values for your scheme derived in problem 1)
- RK-4

(a) On a single plot, for each of these four methods:
Find the value of $y(2)$ and show nth-order convergence of the error on a log-log plot of error versus h.

(b) Pick some fixed value of the error from (a), say $\epsilon = 10^{-6}$, and for each RK-k method, find h_k that yields this error.
Then, have your program count the number of times $f(t, y)$ that had to be evaluated by each method at this value of h_k.
Which method requires the smallest number of $f(t, y)$ evaluations?