Homework 6: Applications of numerical ODE IVP

Due: *Thursday, February 26, 2009*

1. Deriving multistep methods. Use the approach described in class to derive the 4th-order Adams-Bashforth scheme (AB-4). Show all steps, you may want to use Maple to calculate the finite difference derivatives that you will need.

2. Using multistep methods. Solve the nonlinear initial value problem for $y(t)$:

$$\frac{dy}{dt} = \sin(ty^2), \quad y(0) = 3$$

Please follow the steps given in reverse order to solve this problem:

(a) Find the value of $y(2)$, show third-order convergence of error on a log-log plot of error vs. h.

(b) Do not waste memory. Points will be taken off for programs that unnecessarily store big arrays of numbers or execute very slowly. Use the minimum amount of memory needed to carry out the calculations.

(c) Your main routine for this problem should input only one number, N, the number of steps, with $h = 2/N$, and it should return one number, the computed value of u_N that approximates $y(2)$. I should be able to type in any positive value for N into your routine and get a solution. Your routine should have a part to deal with the "start-up problem" and a main loop to calculate AB-3.

(d) The Start-up problem. There is a difficulty in "starting" the multistep method because at $t = 0$ you have only one initial condition, not the "three levels of history" that AB-3 needs. Use Forward Euler to generate enough "starting history" for you to use AB-3 for all the later steps.

Q: Isn't Forward Euler only first-order accurate? Won't the errors I make on the beginning steps contaminate the overall answer and degrade the rate of convergence?

A: Ehh, hmmm, yes, I stand corrected. You need to generate "starting-steps" for AB-3 that will not degrade its accuracy, but first-order accurate Forward Euler is the only non-multi-step method that is explicit and that you've learned so far that will work for this ODE. So,...

Use Forward Euler with Richardson extrapolation to generate sufficiently accurate "starting steps."

(e) Use the third-order Adams-Bashforth (AB-3) multistep method:

$$u_{n+1} = u_n + \frac{h}{12}\left(23f_n - 16f_{n-1} + 5f_{n-2}\right)$$