Homework 4: Numerical Integration and Quadrature

Due: *Thursday, February 12, 2009*

1. **Simpson's rule.** Burden and Faires Chapter 4.3, Part (d) of Questions 5 and 7.

2. **Composite integration.** Let

$$f(x) = \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-s^2}\, ds$$

You will be computing $f(2)$ using the program you previously wrote for trapezoid rule. You can check your results and estimate error by means of comparisons with published tables, or by making use of the alternating series that one gets by expanding the integrand in its Maclaurin series and integrating them term by term. In an alternating series, one knows from elementary calculus that the magnitude of the error is less than the magnitude of the first term of the series that is neglected. The alternating series is

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)n!}$$

   (a) Use the composite trapezoid rule and its error formula discussed in class to estimate $f(2)$ with error magnitude less than 10^{-8}. How many subintervals do you need?

   (b) Let n be the number of subintervals used in part (a). Obtain plot of the base-ten logarithm of an estimate of the magnitude of the error, as a function of the base-ten logarithm of n. Relate the resulting plots to the expected orders of convergence and to the asymptotic error formulas. For sufficiently large n the effects of floating point arithmetic may become apparent.

   (c) Use Romberg integration (based on the trapezoid rule) to estimate $f(2)$ with error magnitude less than 10^{-8}. Write an efficient program to perform the integration. How many subintervals are needed to obtain your estimate? Compare this number of subintervals with the number of subintervals used in part (a).

3. **Gaussian quadrature.**

   (a) Calculate $f(2)$ in problem 2 using the $n = 5$ Gauss-Legendre integration formula and estimate your error.

   (b) How would you compare the accuracy of the trapezoid rule and the Gauss quadrature, given the same number of spatial points?