

Math 107: Linear Algebra and Differential Equations

Practice Test #2

Name: _____

Thursday, November 19, 2009

Lecture section: 107.0_____ Recitation section: 107R.0_____

All answers must be justified. No calculator is allowed.

Question 1. Find an orthonormal basis for the space spanned by the vectors

$$\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}$$

Question 2. Find the general solution of

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 8xe^{2x} \cos x$$

Question 3. A 2-kg mass stretches a spring 1 m. This mass is hung vertically on the spring and then a shock absorber is attached that exerts a resistance of 14 kg/s to the motion. The mass is pulled down 3 m and then released.

(a) Determine the motion of the mass.

(b) Determine the motion of the mass if an external force of $6e^{-2t}$ N is applied to the mass-spring system above.

Question 4. Find all the eigenvalues of

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

And for each eigenvalue, find a basis for the corresponding eigenspace.

Question 5. Consider the linear transformation $T : P_2 \rightarrow P_2$ defined by

$$T(ax^2 + bx + c) = (a + b + c)x^2 + (2a + 3b + c)x + (3a + 5b + c).$$

Find the kernel and range of T . Find their dimensions.

Question 6. If T is defined on \mathfrak{R}^2 by

$$T(x_1, x_2) = (x_1 + 3x_2, 4x_1 + 2x_2)$$

find the matrix A of T relative to the basis

$$\beta = \left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \end{bmatrix} \right\}$$