

Math 107: Linear Algebra and Differential Equations

Mock Exam #1

Name: _____

Thursday, October 1, 2009

All answers must be justified.

Question 1. Compute the determinant and the inverse matrix of

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Question 2. The $n \times n$ matrix A can be turned into the identity matrix by swapping rows 3 and 4, multiplying row 2 by -3 , subtracting row 5 from both rows 1 and 2, swapping rows 2 and 3, and then multiplying row 1 by $1/7$ and row 3 by $2/3$. What is the determinant of A ?

Question 3. For what values of k and h is the following system consistent?

$$\begin{aligned}2x_1 - x_2 &= h \\ -6x_1 + 3x_2 &= k\end{aligned}$$

Question 4. Determine by inspection if the given set is linearly dependent. Explain why.

$$(a). \begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 9 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix}$$

$$(a). \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 8 \end{bmatrix}$$

$$(a). \begin{bmatrix} -2 \\ 4 \\ 6 \\ 10 \end{bmatrix}, \begin{bmatrix} 3 \\ -6 \\ -9 \\ 15 \end{bmatrix}$$

Question 5. Let

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}, \quad w = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

(a) Is w in $\{v_1, v_2, v_3\}$? How many vectors are in $\{v_1, v_2, v_3\}$?

(b) How many vectors are in $\text{span}\{v_1, v_2, v_3\}$?

(c) Is w in the subspace spanned by $\{v_1, v_2, v_3\}$?

Question 6. Let

$$A = \begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix}, \quad u = \begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix}, \quad v = \begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}$$

(a) Determine if u is in the null space of A . Could u be in the column space of A ?

(b) Determine if v is in the column space of A . Could v be in the null space of A ?

Question 7. Let H be the set of points inside and on the unit circle in the xy -plane. That is, let

$$H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 + y^2 \leq 1 \right\}$$

Find a specific example—two vectors, or a vector and a scalar—to show that H is not a subspace of \mathbb{R}^2 .