

Math 107: Linear Algebra and Differential Equations

Practice final exam

Name: _____

Friday, December 11, 2009

Lecture section: 107.0_____ Recitation section: 107R.0_____

All answers must be justified. No calculator is allowed.

Question 1. Find conditions on a, b, c so that $v = (a, b, c)$ in \mathfrak{R}^3 belongs to $W = \text{span}(u_1, u_2, u_3)$, where

$$u_1 = (1, 2, 0), \quad u_2 = (-1, 1, 2), \quad u_3 = (3, 0, -4)$$

Question 2. Find the general solution of the system

$$x_1' = 2x_2 + 2x_3$$

$$x_2' = 2x_1 + 2x_3$$

$$x_3' = 2x_1 + 2x_2$$

Question 3. Find a basis and dimension of the subspace W of \mathfrak{R}^3 where

(a) $W = \{(a, b, c) : a + b + c = 0\}$

(a) $W = \{(a, b, c) : a = b = c\}$

Question 4. Solve the initial value problem $x' = Ax$ with

$$A = \begin{bmatrix} -2 & -2.5 \\ 10 & -2 \end{bmatrix}, \quad x(0) = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

Question 5. Let $V = P(t)$, the vector space of real polynomials. Determine whether or not W is a subspace of V . Explain.

(a) W consists of all polynomials with integer coefficients.

(b) W consists of all polynomials with degree ≥ 6 and the zero polynomial.

(c) W consists of all polynomials with only even powers of t .

Question 6. (a) Find the matrix of fundamental solutions for the homogeneous system

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

(b) Find the general solution for the nonhomogeneous system

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e^t \\ e^{-t} \end{bmatrix}$$

Question 7. (a) Find an invertible matrix P and a diagonal matrix D such that

$$A = PDP^{-1}, \quad A = \begin{bmatrix} 6 & -2 \\ 6 & -1 \end{bmatrix}$$

(b) Find A^{-1} without directly inverting A .

Question 8. Let S consist of the following vectors in \mathfrak{R}^4 :

$$u_1 = (1, 1, 0, -1), \quad u_2 = (1, 2, 1, 3), \quad u_3 = (1, 1, -9, 2), \quad u_4(16, -13, 1, 3)$$

(a) Show that S is orthogonal and a basis of \mathfrak{R}^4 .

(b) Find the coordinates of an arbitrary vector $v = (a, b, c, d)$ in \mathfrak{R}^4 relative to the basis S .