Math 263 - Computational Topology

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Course: Math 263
Course Hours: M,W 4:10-5:25
Title: Computational Topology
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Abstract:

The phrase Computational Topology can be interpreted in a variety ways. It might refer to the use of computers to compute invariants of topological spaces that Mathematicians want to know, like homology groups and things like that. On the other hand it might refer to the use of methods and ideas developed in topology to improve the computations that come up in various parts of science, like water-flow on terrains, segmentation of medical image datasets, protein interfaces, etc. Or it might be a hybrid of the two in which we do the former when we see a good application to the latter, recognizing that different applications often require radically different approaches. It is this last point of view that I will take in this mini-course.

So a short definition might be that *Computational Topology is the development of algorithmic tools implementing topological concepts for use in the sciences and engineering.* This is a recent subject of much interest in computer science where Computational Topology is growing out of Computational Geometry, a more established field (by CS standards, meaning it is more than 5 years old!). One of the main reasons is that topological issues continually arise when you play with large data sets. Smoothing and looking at data at multiple scales has mostly been seen as a local process, with the inevitable result that small local errors tend to accumulate into large global ones. And its this global perspective that distinguishes topology.

Here are the goals we'll try to meet in the course:

Course Goals

- Finding Topological Structure in Data Sets
- Constructing hierarchical representations of noisy 2d and 3d data.
- Characterization of Shape
- Applying Topological Methods to Specific Problems including:
    1. Terrain Structure, Visibility and Trafficability
    2. Characterization Molecular Shape
    3. Medical Image Segmentation
    4. Gene Regulatory Network Topology

Approximate Course Outline:

1. Quick Review of Topology
    (a) Topological Spaces, Metric Spaces, Continuous Maps and Homeomorphisms
    (b) Simplicial Complexes
    (c) CW-Complexes
    (d) Simplicial Homology, Relative Homology
    (e) Singular Homology and its equivalence with Simplicial Homology
    (f) Exact Sequences and the Snake Lemma
    (g) Exact Sequence of a Pair
    (h) Cellular Homology
    (i) Mayer-Vietoris
    (j) Cohomology
    (k) Manifolds and the Fundamental Class
    (l) Poincaré Duality, Lefschetz Duality and Alexander Duality
2. Review of Algorithms
3. Algoritms for Computing Homology
    (a) Smith Normal Form
    (b) Others
4. Persistence
    (a) Normal
    (b) Dynamic

(c) Stability

5. Triangulating Data Sets
   - Voronoi and Delaunay
   - α-Shapes
   - Witness Complexes
   - WRAP
6. Computational Morse Theory
   - Morse Complex
   - Segmentations
   - Morse-Smale Complex
   - Persistence
   - Moving View Frames
7. Applications to Terrains
   - Watershed algorithms
   - Incorporating Geological Information
   - Moving View Frames for Terrains
   - Visibility
   - Trafficability
8. Applications to Medical Imaging
   - 2d and 3d Segmentation Methods
   - The Morse Theory Approach
   - Relationship to Deformable Models (Snakes)
9. Applications to Protein Structure
   (a) Interface Surfaces
   (b) Morse Theory of Energy Potentials
   (c) Morse Theory of Distance Function
10. Applications to Shape Description
11. Applications to Gene Regulatory Networks

Some References:

- Algorithms:

  1. Aho, Hopcroft, Ullman, The Design and Analysis of Computer Algorithms, Addison-Wesley, 1974.
  2. Cormen, Leiserson, Rivest, and Stein. Introduction to Algorithms, 2nd Ed., McGraw Hill, Boston, 2001.
  3. Robert Tarjan. Data Structures and Network Algorithms. SIAM, 1983.

- Computational Geometry:

  1. Mark de Berg, Otfried Schwarzkopf, Marc van Kreveld, and Mark Overmars. Computational Geometry. Algorithms and Applications. Springer-Verlag, 1997.

- Computational Topology:

  1. Herbert Edelsbrunner. Geometry and Topology for Mesh Generation. Cambridge Univ. Press, 2001.
  2. IMA New Directions Short Course: http://www.ima.umn.edu/new-directions/2004NDshort-course/

- Topology:

  1. James Munkres. Elements of Algebraic Topology. Addison Wesley, 1984.