Spring 2006

MTH 103: Answers to Even Problems

LESSON 2 (12.2, p. 786): Vectors in R^3

18. $\text{comp}_a b = \text{comp}_b a = -11/\sqrt{14}$ 42. perpendicular   62. $\cos^{-1}(1/\sqrt{3})$

68. $x - y + 2z = 8$ (plane through A and $\perp$ to **n**)   70. $\alpha = \cos^{-1}(-1/3)$

LESSON 3 (12.3, p. 794): Cross product

30. $16\sqrt{195}/195$

LESSON 4 (12.4, p. 801): Lines and planes

14. $x = 2 + 3t$
$y = -1 + t$ $-\infty < t < \infty$; $\dfrac{x-2}{3} = \dfrac{y+1}{1} = \dfrac{z-5}{-1}$
$z = 5 - t$

30. $x + y - 2z = -2$ 32. $15x - 9y - z = 16$

40. $\pi/2$ 56. $(7\sqrt{2})/5$

LESSON 5 (12.5, p. 813): Curves and motion in R^3

p. 813 2. Fig. 12.5.18   4. Fig. 12.5.15   46. $\alpha = \pi/4$

64. $(6 \sin 15°)^2 \approx 2.4115427$ ft. (*Hint*: See Example 10 in text.)

p. 845   16. $x + 2y + 3z = 6$

LESSON 6 (12.6, p. 828): Curvature and acceleration

6. $L = 2e - (1/e) - 1$ 10. $\kappa = \sqrt{2}/250$ 18. $\mathbf{T} = 1/\sqrt{13}\,[3\mathbf{i} - 2\mathbf{j}]$, $\mathbf{N} = 1/\sqrt{13}\,[-2\mathbf{i} - 3\mathbf{j}]$

34. $\dfrac{2\sqrt{9t^4 + 9t^2 + 1}}{(9t^4 + 4t^2 + 1)^{3/2}}$ 42. $\mathbf{T} = 1/\sqrt{14}\,[\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}]$, $\mathbf{N} = 1/\sqrt{266}\,[-11\mathbf{i} - 8\mathbf{j} + 9\mathbf{k}]$

54. $\kappa = 8\sqrt{2}/3$; center of curvature is (21/16, 21/16)

LESSON 7 (12.7, p. 837): Cylinders and surfaces

30. ellipsoid with center at (0,0,0)

LESSON 8 (13/2, p. 857; 13/3, p. 866): Functions of several variables; limits and continuity

p. 857 54. 13.2.39 56. 13.2.40 58. 13.2.43

p. 866 24. does not exist 30. $-\pi/2$

LESSON 9 (13.4, p. 875): Partial derivatives

58. (a),(c)

LESSON 10 (13.5, p. 886): Maximum-minimum

12. (0,0,0), (1,2,4/e), (−1,2,−4/e), (1,−2,−4/e), (−1,−2,4/e) 18. (−1,0,−1)

24. 3 is the maximum value and −1/4 is the minimum value

26. 2 is the maximum value and −1/2 is the minimum value

28. 2 is the maximum value and −2 is the minimum value

32. P(2,2,2) 38. base 20 ×20, height 10

LESSON 11 (13.6, p. 895): Increments and differentials

18. f(Q) ≈ 12 + 31/120 26. $\dfrac{135{,}817}{4{,}320}$

34. $dV = \pm(6.3)\pi\ \text{cm}^3$, $dS = \pm(3.6)\pi\ \text{cm}^2$ 42. S ≈ .875, S = .884

LESSON 12 (13.7, p. 904): Chain rule

28. $\dfrac{\partial w}{\partial x} = \dfrac{y[y^2 - 3x^2 - (x^2+y^2)^3]}{(x^2+y^2)^3}$, $\dfrac{\partial w}{\partial y} = \dfrac{x[x^2 - 3y^2 - (x^2+y^2)^3]}{(x^2+y^2)^3}$ 37. 14/3 L./min.

LESSON 13 (13/8, p. 915): Directional derivative and the gradient

6. $\nabla f(12,3,4) = (12/13)\mathbf{i} + (3/13)\mathbf{j} + (4/13)\mathbf{k}$

8. $\nabla f(2,1,0) = 4\mathbf{i} + 0\mathbf{j} - 3\mathbf{k}$

14. $D_{\mathbf{u}}f(-3,3) = -7/30$

26. direction: $(1/\sqrt{3})(\mathbf{i} - \mathbf{j} - \mathbf{k})$;
max. directional derivative = $\sqrt{3}$

28. direction: $(1/\sqrt{14})(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$;
max. directional derivative = $\sqrt{14}/8$

30. $4x + 9y = 35$

34. $4x + 23y + 17z = 25$

48. (a) $-170/13$; (b) $(1/10\sqrt{2})(-6\mathbf{i} + 8\mathbf{j} - 10\mathbf{k})$; $10\sqrt{2}$

50. $16x - 12y - 4z = 11$

56. (a) $\sqrt{2}/10$; $\tan^{-1}(\sqrt{2}/10)$; (b) $7\sqrt{2}/10$; $\tan^{-1}(7\sqrt{2}/10)$

LESSON 14 (13.9, p. 924): Lagrange multipliers

10. maximum value = $\sqrt{3}/9$, minimum value = $-\sqrt{3}/9$

30. base 20×20, height 10

42. maximum value = $(9 + 3\sqrt{5})/4$, minimum value = $(9 - 3\sqrt{5})/4$

LESSON 15 (13.10, p. 933): Second derivative test

4. saddle point at (2,–3)

6. saddle point at (6,–4)

8. local minimum at (1,1), saddle point at (–1/3, –1/3)

10. saddle point at (0,0), local maximum at (1,1).

12. saddle point at (0,0), local minimum at (2,–2)

20. local minimum at (2,1), saddle point at (2,–1), saddle point at (–1,1), local maximum at (–1, –1).

LESSON 16 (14/1, p. 945; 14/2, 953): Double integrals

p. 945 32. 4 34. 2

p. 953 12. $\pi/4$ 18. 0 22. 1/8

30. $(e - 1)/2e$ 42. 1/3

LESSON 17 (14.3, p. 959): Area and volume

18. 9/10

22. $\int_{-2}^{1}\int_{x}^{2-x^2}(1+x^2+y^2)\,dydx$ =837/70

24. 1427/420

28. $\int_{0}^{2}\int_{y/2}^{(4-y)/2}(8-4x-2y)\,dxdy$ = 16/3

30. $\int_{-3}^{5}\int_{(x-5)/2}^{(5-x)/2}(25-x^2-y^2)\,dydx$ = 1792/3

LESSON 21 (10.2, p. 635): Polar coordinates

2d. $(2,2\pi/3)$, $(-2,-\pi/3)$ 6. $r = 5$ 24. $(x-3)^2 + y^2 = 9$, $r = 6\cos\theta$

56. $(0,0)$, $((2-\sqrt{2})/2, 3\pi/4)$, $((2+\sqrt{2})/2, -\pi/4)$

LESSON 22 (14.4, p. 966): Double integrals in polar coordinates

p. 966

2. $9\pi/4$ 10. $3\pi/2$ 12. $\pi a^4/2$

14. $\pi(2-\sqrt{3})/2$ 28. $\pi/2$ 38. 12π

p. 959

34. $\frac{2\pi}{3}[2\sqrt{2}-7/4]$ 42. 24π

LESSON 18 (14.5, p. 975): Applications

8. $\bar{x} = 0$, $\bar{y} = 27/5$ 42. $\bar{x} = \bar{y} = 4r/3\pi$ 44. $\bar{x} = \bar{y} = 2r/\pi$

LESSON 19 (14/6, p. 985): Triple integrals

2. $\pi^4/2$

6. $\int_0^3 \int_0^{2-(2/3)x} \int_0^{6-2x-3y} (2x+3y)\, dz dy dx$

8. 512/15

10. 512/3

12. 128/5

14. 1/6

22. $\bar{x} = \bar{y} = 0$, $\bar{z} = 3R/8$

28. $\pi HR^4/2$

LESSON 23 (12.8, p. 843): Cylindrical and spherical coordinates

26. sphere of radius 5, center (0,0,0)

30. negative z-axis

55b. $\csc\phi \le \rho \le 2,\ \pi/6 \le \phi \le 5\pi/6,\ 0 \le \theta \le 2\pi$

LESSON 24 (14.7, p. 993): Triple integrals in cyl/sph coordinates

4. $(8/15)\pi a^5$

20. $z = Hr/R,\ 0 \le r \le R$

22. $M = \pi k a^4/4$; (0,0,8a/15)

26. $(8 - 5\sqrt{2})\pi a^5/30$

30. $8\pi/3$

38. 2π

LESSON 20 (14.9, p. 1007): Change of Variables

4. $-1/8[u^2 - v^2]^{-1/2}$

6. $4/(u^2 + v^2)^2$

8. area = ½ *ln* 2

10. 1/8

12. 1/8 .

LESSON 25 (14.8, p. 1000): Surface area

2. 1

LESSON 26 (15.1, p. 1018): Vector fields

12. figure 15.1.9

20. div **F** = 12, curl **F** = 2**i** + 3**j** + **k**

LESSON 27 (15.2, p. 1028): Line integrals

2. $(5\sqrt{5} - 1)/12$

4. 1

6. 69/4

10. −9

12. −2

14. 71

16. $7\sqrt{22}$

22. $90k\pi$

36. 2π

LESSON 28 (15.3, p. 1036): Independence of path

2. $f(x,y) = 2x^2 - xy + 3y^2$ 24. -2 26. $(1/e) - e$

28. $f(x,y,z) = x^2 - xy - xz + y^2 + z^2$ 30. $\pi, -\pi$; NO

LESSON 29 (15.4, p. 1045): Green's Theorem

2. $-2/3$ 16. $1/12$ 18. 0 22. $\pi 3^5/2$

34. $4/3$

LESSON 30 (15.5, p. 1055): Surface Integrals

p.1055

2. $\sqrt{14}\int_0^3 \int_0^{2(3-x)/3} [6xy - 2x^2y - 3xy^2]\, dydx$

6. $\pi/2$ 10. $609\pi\sqrt{2}/2$ 14. $27/8$ 18. 44π

p. 1072

18. $4\pi a^5$

LESSON 31 (15.6, p. 1063): Divergence Theorem

4. 24 6. 48 8. $31{,}250\pi/3$

LESSON 32 (15.7, p. 1070): Stokes' Theorem

2. 4π 10. -2π 14. $f(x,y,z) = \dfrac{(x^2+y^2+z^2)^{5/2}}{5}$

LESSON 5 (10.4, p. 650; 10.5, p. 657): Parametric curves in R^2

p. 650

16. $y = 1 - x^2, -1 \le x \le 1$
[(−1,0) to (1,0) back to (−1,0)]

18. (a) $x + y = \sqrt{2}/2$; (b) concave up

25(b). At (3,0), there are two tangents. When $t = \sqrt{3}$, slope = $\sqrt{3}$. When $t = -\sqrt{3}$, slope = $-\sqrt{3}$.

30. $x = \dfrac{3t}{1+t^3}$ and $y = \dfrac{3t^2}{1+t^3}$, $0 \le t < \infty$

p. 657

12. $(2\sqrt{2} - 1)/3$

14. $\sqrt{2}\,(e^{\pi} - 1)$

34. 36