

Monday, May 20th

9:00–9:40 Globally Coupled Oscillator Networks

Philip Holmes, Eric Brown, Jeff Moehlis

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I will describe joint work with Eric Brown and Jeff Moehlis on globally-coupled, phase oscillator networks on N -dimensional tori. We focus on the effects of symmetry and the forms of the coupling functions, derived from underlying Hodgkin-Huxley type neuron models, on existence, stability, and degeneracy of phase-locked solutions in which subgroups of oscillators share common phases. Implications for stochastically forced networks, and ones with random natural frequencies, are discussed. We indicate an application to modeling the brain structure locus coeruleus: an organ involved in cognitive control.

9:40–10:20 Statistical Characterization of Disordered Materials

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It is a fundamental challenge to understand the properties of disordered materials that are robust in that they do not depend on how the material is prepared or on details of the disorder and interactions. This talk will discuss our work on the statistics of force inhomogeneities in stationary granular materials (which exhibit some robust features) and on the properties of the ground states of the two-dimensional nearest-neighbor $\pm J$ spin glass (which exhibit remarkable variability).

10:35–10:55 Breakup of Viscoelastic Liquid jets under Surface Tension

Michael Renardy

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It is well known that the addition of polymers to liquid jets can suppress or at least significantly delay the breakup of jets. The lecture will review recent results on one-dimensional model equations for jets of viscoelastic liquids with a variety of constitutive models. The mathematical questions considered are:

1. Does breakup in finite time occur under the influence of surface tension?
2. Can finite time breakup occur in a filament that is being stretched even in the absence of surface tension?
3. How can the approach to breakup be described asymptotically?

11:00–11:20 Equi-integrability Results for 3D-2D Reduction Problems

Marian Bocea

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We show that the Dirichlet problem on an arbitrarily large cylinder for fixed affine lateral boundary conditions admits p -equi-integrable minimizing sequences energetically preferring thinner and thinner reference domains. The proof is based on a decomposition result in the 3D-2D asymptotic analysis of thin films, recently obtained by the authors.

11:25–11:45 Global Bifurcation Branches of Neumann Boundary Value Problems

Junping Shi

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Semilinear elliptic equations with Neumann boundary conditions arise in many models of mathematical biology and material sciences. In this talk, we consider

$$\Delta u + \lambda f(u) = 0, \quad x \in \Omega, \quad \partial u / \partial n = 0, \quad x \in \partial \Omega,$$

where Ω is a rectangle $(0, a) \times (0, b)$ in \mathbf{R}^2 . For balanced and unbalanced f , we obtain partial descriptions of global bifurcation diagrams in (λ, u) space. In particular, we rigorously prove the existence of secondary bifurcation branches from the semi-trivial solutions, and obtain mushroom-like and tree-like global bifurcation diagrams for different models.

1:10–1:50 Architecture and Dynamics of the Primary Visual Cortex

David W. McLaughlin

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1:50–2:30 Faraday Wave Pattern Formation

Mary Silber

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Standing surface waves form on a layer of fluid when it is subjected to a sufficiently strong periodic acceleration in the vertical direction. There have been numerous theoretical and experimental investigations of pattern formation in this so-called Faraday wave system in the last 15 years or so, in part because it exhibits patterns not seen in other systems (rhombic and triangular patterns, quasipatterns, superlattice patterns, localized structures akin to oscillons, etc.). Hence it provides a versatile framework in which to develop and test our general understanding of the nonlinear pattern formation process. However, this versatility comes at some cost. Specifically, many of the more exotic patterns are only realized when the periodic forcing function contains more than one frequency component; the introduction of nonsinusoidal periodic forcing functions leads to a large control parameter space that is difficult to fully characterize. For example, there is an increase from 2 parameters (frequency and amplitude) to 5 parameters in the next simplest case of two-frequency forcing (frequencies, amplitudes and relative phase). I will report on recent work with Jeff Porter and Chad Topaz that helps explain the role of each of these parameters in the pattern formation process. Our analysis highlights the effects of weakly broken time translation and time reversal symmetries that result in the weak forcing and damping case. The approach should carry over to other pattern-forming systems that support standing waves. It also suggests how one might design forcing functions to control pattern selection.

2:40–3:20 Resonant Patterns in a Chemical Reaction-Diffusion System

Anna L. Lin

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Frequency locking has been well studied for driven nonlinear oscillators, yet little is known about the shape and content of the Arnol'd tongues of locking for spatially extended systems. We present experiments which examine the effect of periodic optical perturbations on a light-sensitive oscillatory chemical reaction-diffusion system. Transitions from spiral traveling wave patterns to various frequency locked standing wave patterns are observed. Numerical simulations of a reaction-diffusion model yield qualitatively similar phenomena. The observed pattern characteristics are consistent with the stable solutions of a forced amplitude equation.

3:40–4:20 Theory and Application of Initial-Boundary-Value Problems for Nonlinear Dispersive Waves

Jerry L. Bona

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This lecture will focus on recent work connected with initial-boundary-value problems for Korteweg-de Vries type equations. Theoretical results suggested by experiments and numerical simulations will be described as well as the application of these ideas in problems arising in coastal engineering.

Tuesday, May 21st

9:00–9:40 Analytic Consequences of Incompressibility

Stuart S. Antman
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Many deformable solids, like rubber and living tissue, are effectively incompressible. This means that the Jacobian determinants of all their deformations must be everywhere equal to 1. This talk will show by simple examples that the equations governing the motion of incompressible bodies are much more complicated than those governing the motion of compressible bodies (whose deformations need only preserve orientation), but have far more regularity. The role of incompressibility will be briefly related to the question of constructing invariant dissipative mechanisms for hyperbolic conservation laws.

9:40–10:20 The Euler-Lagrange Equation and Minimizers in Elastostatics

John M. Ball
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One-dimensional examples from the calculus of variations show that it is not always the case that minimizers are weak solutions of the corresponding Euler-Lagrange equation. Whether or not this is the case for nonlinear elasticity is an open problem. The lecture will revisit old work of the speaker, giving somewhat improved results concerning conditions under which certain weak forms of the equilibrium equations can be established. Some other outstanding open questions concerning regularity in elastostatics will be reviewed.

10:35–10:55 Directed Force Chain Networks and Stresses in Granular Materials

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Force chains are prominent features in the stress patterns observed in experiments and simulations of granular materials. Bypassing the notoriously difficult problem of understanding the origin of the chains, we develop a theory that takes the chains themselves to be primary physical quantities. We construct a "Boltzmann" equation describing the densities of splitting and fusing chains to form a directed force chain network (DFCN). It turns out that nonlinear terms in the equation cannot be neglected and the properties of the DFCN are generally dominated by a nontrivial, anisotropic fixed point. Some interesting exact results have been obtained for 2D systems with 6-fold and 8-fold orientational order. Much remains, however, for Dave to figure out.

10:55–11:15 Instability of Local Deformations of an Elastic Rod

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We study the instability of pulse solutions of two coupled nonlinear Klein-Gordon equations by means of Evans function techniques. The system of two coupled Klein-Gordon equations describe the dynamics of a three-dimensional elastic rod. We determine a condition on the speed of the traveling pulse which ensures instability.

11:20–11:40 Solid Cavitation

M. Ben Amar, M. Brenner and Jim Rice
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By focusing a high intensity acoustic wave in liquid helium at low temperature (65mk) Balibar and coworkers have observed the formation of a solid nucleus when the background pressure oscillates around the melting pressure. The experimentalists measure the growth of the solid as a function of time. We derive the equation of motion for the solid-liquid interface, including the discontinuity conditions expressing conservation of mass, momentum and energy and non-negative entropy production. Quantitative agreement requires including the elastic response of the solid. In this talk, I will stress the similarities and differences between this process and the cavitation of bubbles, especially the Rayleigh collapse.

11:40–12:00 Effective Acoustic Models of Porous Media

R. P. Gilbert, *A. Panchenko*
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We develop a homogenized effective model for propagation of acoustic waves in a fluid-saturated solid with microstructure. The fluid is described by the compressible Navier-Stokes equations, and the solid is linear and elastic. We assume that the solid phase is connected, and the typical size of the pores ϵ is small compared to the size of the specimen. We show that as ϵ tends to zero, the composite behaves as a single-phase linear viscoelastic medium. We use rather flexible geometric assumptions which hold true for many disordered materials occurring in nature.

12:00–12:20 Wave Equations under Strong Constraining Forces

Jalal Shatah and *Chongchun Zeng*
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In this talk, we consider wave equations of $u(t, x) \in R^n$ where there is a strong constraining potential. This potential achieves its minimum on a submanifold $M \subset R^n$. We study the limit of solutions with $u(0, x) \in M$ as the constraining potential tends to infinity. On the other hand, this problem can also be viewed as realizing a homomorphic constraint (to a submanifold in the configuration space) to a Hamiltonian motion by using a strong constraining potential.

1:20–2:10 The Axially-Compressed Cylinder: A Exemplar for Buckling and Bifurcation

G. W. Hunt

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The classical buckling problem of the long, thin, axially-compressed cylindrical shell has certainly proved one of the most stimulating intellectual challenges in the history of mechanics, and is yet to be fully understood. Thoroughly nonlinear and with a deceptively simple geometry, it has taxed many from von Karman onwards. It played a leading role in Koiter's early classification of bifurcation phenomena, and more recently has been the main instrument in the identification of the hidden symmetry problem. Much of its complexity arises from the occurrence of multiple bifurcation points, with modes that want to combine in ways that break the inevitable symmetry of a single mode response.

It has had yet another heuristic role to play, however, as an exemplar to underline the fundamental differences between localized and distributed responses. In the absence of internal pressure, the buckle pattern that is found experimentally is periodic in the circumferential direction, but localized axially. During pre-buckling, the system locks into a circumferential wavenumber that is maintained until well into the post-buckling range. In contrast, in the axial direction the response localizes early and completely, at load levels commensurate with those of experimental collapse, to a buckle pattern that bears no relation to any linear prediction.

The talk will be built around the hypothesis that the circumferential wavenumber of initial buckling for an axially-compressed cylindrical shell of moderate length can be predicted from the classical (Koiter circle) linearized result, by assuming that the small-deflection buckle pattern will adopt either one or two half-waves longitudinally. Except for the preservation of the circumferential wavenumber, subsequent axial localization will quickly eliminate any similarity with this linear eigenvalue result. Yet the circumferential wavelength prediction is important, in that it has the potential to untangle an otherwise complex web of post-buckling solutions into a single sequence of cellular buckles, with localized cells buckling in turn accompanied by an oscillation in load between two different levels. In practice, such a sequence is likely to be broken up by a sequence of mode jumps to similar patterns, each jump reducing the circumferential wavenumber by a count of one.

Checks against physical and numerical experiments, both by direct wavenumber comparisons and via a scaling law, provide strong evidence that the hypothesis is correct. The conclusions lend weight to the argument that initial buckling is governed more by the position of the classically-defined Maxwell load, than the conventional critical load based on linear theory.

2:10–2:50 A Model of Blood Flow through the Abdominal Aorta after Endovascular Repair

Suncica Canic

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Endovascular prostheses are used in the treatment of various cardiovascular diseases (aneurysm, vascular lesions, stenosis, occlusions) to either replace or repair failing blood vessels. After every cardiovascular surgery or endovascular repair during which an endovascular prosthesis has been inserted in the aorta, the wall properties of the compliant vessel change in the region where the prosthesis is placed. In this talk I will present a study of the effects of the rapidly changing elastic wall properties on the wall deformations and on the blood flow.

The study is based on a one-dimensional (Navier-Stokes) model of blood flow through an axisymmetric compliant vessel whose elastic properties change abruptly in the axial direction due to an inserted endovascular prosthesis. The model equations are in the form of a quasilinear system of hyperbolic PDEs with discontinuous coefficients. I will comment on various mathematical and numerical difficulties associated with the analysis and numerical simulation of such equations and present an investigation which takes into account transmural pressure and wall deformations to design an optimal strategy in the use of multiple overlapping stents (Wallstent stents) in the endovascular treatment of abdominal aortic aneurysm.

Collaborators: -Dr. Ravi-Chandar, Mechanical Engineering at UT Austin, -Cardiologist Dr. Zvonko Kra-

jeer, M.D., St. Lukes' Hospital and the Texas Heart Institute, Houston, and -Dr. D. Mirkovic, Computer Science, University of Houston -Dr. Eun Heui Kim, Mathematics, University of Houston.

3:00–3:40 Growth and Dynamics of filamentous microorganisms

Alain Goriely

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Most filamentous microorganisms belong to two groups, the eukaryotic filamentous fungi, and the prokaryotic filamentous actinomycetes. The cellular structures observed in these two groups are fundamentally different. However, they exhibit similar morphologies, growth patterns, and mycelial and aerial growth forms. Despite some earlier geometric models there is no biomechanical understanding of filamentous growth. In this talk, I will present recent experimental results and new models for the vegetative growth of filamentous bacteria. In particular, I will develop a theory of growing biomembranes to model tip growth.

9:00–9:40 Averaging in Temporally Varying Flows and the Homogenization of Gravity Currents in Porous Media

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We discuss enhanced mixing induced by complex fluid motion, overviewing the importance of these problems from a general perspective in modeling turbulence with closure coefficients, and then focusing upon rigorous averaging theories in idealized contexts to try to explicitly quantify effective mixing coefficients.

We consider the idealized problem of calculating enhanced diffusivities for passive transport in steady periodic geometries, reviewing the poor dependence of these coefficients upon large scale flow parameters. Through the introduction of temporal variation into these models through rapid wind fluctuations, we present a theory which identifies regions in the Peclet-Strouhal plane for which fluctuation massages the poor coefficient dependence existing in the steady geometry, and regions with the mixing coefficients plagued by non-monotonic Peclet dependence. In joint work with Bonn, Camassa, McLaughlin and McLaughlin, we present two parameter joint asymptotics identifying a variety of Peclet-Strouhal scaling behaviors for an explicit class of shear models.

Lastly, we turn to the averaging of slumping gravity currents in porous media, a moving interface problem with spatially dependent medium coefficients, and explore slumping rates and spatial interface profiles through homogenized averaging.

9:40–10:20 Simulation-Based Risk Management of Data for Rapid Mass Flows

E. Bruce Pitman
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Our research effort integrates mathematical modeling, high performance computing and database management, for hazard mitigation of rapid mass-flows at volcanoes. This program contains three main thrusts: (1) developing realistic simulations of geophysical mass flows; (2) integrating data from several sources, including simulation results, remote sensing data, and GIS data, and (3) extracting, and organizing information in a range of formats and fidelity, including audio, visual, and text, to scientists and decision-makers involved in risk management. In this talk, we focus on aspects of modeling and computing. Pioneering work of Savage and his colleagues formulated a model of debris flow and rock avalanches that is analogous to the shallow water system, the principal difference being the momentum source terms in the model. To date, numerical simulations of the governing equations have considered flow over a relatively simple topography. Here we present a parallel, adaptive grid simulation framework to solve model system. Our simulations incorporate topographical elevation data from specific volcanoes that we study. We also describe a data management scheme that facilitates processor communication and data retrieval.

10:35–10:55 The Use of Computers in Proving Theorems in Differential Equations

Joseph D. Fehribach
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Over the past thirty years or so, the ability of computers to contribute in the proving of mathematical results in differential equations and in other disciplines has increased significantly. Examples of the use of computers in proving results from Stokes flow, hyperbolic PDE and reaction-diffusion-conduction problems will be presented.

10:55–11:15 Kinematics of Growing Curves: Models of Fungal Hyphae

Gyorgy Karolyi, Alain Goriely, Michael Tabor

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Growth of fungal hypha occurs at its tip, but in spite of efforts the shape of the tip and the mechanism maintaining constant tip shape are still unknown. Most previous attempts model the tip as a surface of revolution, dealing with a two-dimensional section of the hypha. We show that these previous studies, obtaining various possible tip shapes, can be discussed in the general framework of the kinematics of growing curves. We point out that many other possible shapes can emerge fulfilling the kinematic requirements for such curves modeling tip growth of fungal hyphae.

11:20–11:40 Central-Upwind Schemes for Systems of Balance Laws

Alexander Kurganov

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We study the application of high-order semi-discrete central-upwind schemes to multidimensional systems of balance laws. These schemes are Godunov-type methods – a piecewise polynomial interpolant, reconstructed from the cell averages computed at time t , is evolved to the next time level, $t + \Delta t$, using a particularly simple spatial discretization combined with a stable ODE solver of an appropriate order. The numerical fluxes are obtained by including the Riemann fans into the control volumes of a (varying) size, determined by one-sided local speeds of propagation, and by passing to the limit as $\Delta t \rightarrow 0$.

The main advantages of central schemes is their simplicity, since no (approximate) Riemann problem solver, characteristic decomposition, operator or dimensional splitting is required. The main challenge in application of central-upwind schemes to balance laws is how to preserve the balance between the numerical fluxes and the source terms.

In the case of Saint-Venant system of non-homogeneous shallow water equations, the goal is achieved by using a special quadrature for the source average. For the compressible Euler equations with source terms due to a static gravitational field, we rewrite the system in terms of a variable, which remains constant at stationary steady states and apply the central-upwind schemes to the new system. A special treatment of a discontinuous source term is required for the Savage-Hutter system for granular materials. The application of the central-upwind scheme to balance laws with a stiff source term (e.g., detonation waves) will be also discussed.

11:40–12:00 Decay of Solutions to Nonlinear Parabolic Equations: Renormalization Group and Analysis Methods

Huseyin Merdan

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Renormalization group methods (RG) have provided a powerful tool for calculation of key exponent that are otherwise extremely difficult to evaluate. This talk presents an application of RG to differential equations. In this talk, we will discuss Renormalization group calculation of anomalous exponents for nonlinear diffusion. In particular, I will present some of our recent results for this problem.

We will consider the heat equation with nonlinearity, $u_t = u_{xx} + \epsilon F(x, u, u_x, u_{xx})$, in a finite domain, where ϵ is a (small) parameter. Using renormalization group approach we will calculate that for large space and time, the solution is characterized by

$$u(x, t) \sim (t^{(-1/2)+\alpha})u(xt^{-1/2}, 1)$$

where alpha is a simple function of powers of x, u, u_x and u_{xx} in F .

At the end, we will verify the result obtained by RG with rigorous methods using transformations for some examples such as $u_t = u_{xx} + \epsilon(x^{-1})u_x$.

1:10–1:50 The Blob Projection Method for fluid/interface computations

Ricardo Cortez and Michael Minion

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The Blob Projection method is a finite difference numerical method for modeling the interaction between flexible elastic filaments and an incompressible fluid. The forces exerted by the filaments on the fluid are modeled through the use of regularizations and the velocity field they induce is computed directly on a regular Cartesian grid via a smoothed dipole potential. Results that illustrate some of the properties of the method and applications to the motion of swimming creatures will be presented.

1:50–2:30 On Various Numerical Issues in Plasticity

Pierre A. Gremaud

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Several computational issues related to the numerical study of plasticity and elastoplasticity problems will be discussed. Although the specific problems to be presented relate to granular materials and soil mechanics, the corresponding numerical issues have implications far beyond those fields. One the main issues will be the numerical treatment of partial differential algebraic equations (PDAEs).

For plasticity problems, algebraic constraints come for instance through the presence of yield conditions. Many other problems present the same type of structure, although generally in a simpler "solvable" form. For instance, in case of the Euler equations of gas dynamics, equations of state can be regarded as algebraic constraints. How to best deal numerically with this kind of problems is not clear in general. Both theory and practice for the study of DAEs (ODEs + algebraic constraints) have recently reached an impressive degree of maturity. However, PDAEs present additional challenges; for instance, the structure of the system itself (conservation form, etc...) may be very important. Preliminary thoughts about those issues will be presented.

2:40–3:20 Singular Shocks in a Two-Fluid Model for Bubbly Flows

Barbara Lee Keyfitz

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In a number of physical and engineering systems, the results of basic modeling give rise to equations of non-hyperbolic type: these are equations whose linearizations can be, in effect, meaningless vis a vis the character of the physical problem at hand. The user community has been uncertain about how to treat these models but a simple mathematical explanation is that the models are not a complete description of the full physical phenomenon. As far as they go, the models predict phenomena which are completely consistent with the physical considerations that went into their derivation.

In this talk, I show how a standard two-fluid model for two phase flows can be analyzed using conservation law theory to predict reasonable phenomena. Novel weak solutions, in the form of singular shocks, appear in the analysis. Better modeling of the flow requires careful analysis of drag terms, and the way in which these terms may interact with the singular shock solutions is an intriguing open problem.

3:40–4:20 Nonlinearly Dispersive Water Waves

Roberto Camassa
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Internal waves are one of the most important phenomena in geophysical fluid dynamics. Surprisingly, this type of wave motion seems to be only partially understood even within the relatively well controlled set-up of wave tank experiments. This talk will discuss the fundamental dynamical assumptions which lead to known models of internal wave propagation in two-fluid systems, and will review the comparisons of these models with available experimental data. It will be shown that the asymptotic expansions based on the weak nonlinearity assumption are inadequate for the majority of dynamical regimes. A derivation of a class of new models to remedy this situation will be sketched. The resulting evolution equations retain the simplicity of known models while striving to maintain the full nonlinearity of the original Euler equations. The general structure of the new models encompasses known weakly nonlinear theories, such as those of Korteweg-de Vries (KdV) and Benjamin-Ono (BO), while presenting new mathematical challenges in the theory of evolution equations. A surprising spin-off of this modeling effort, under appropriate restrictions, is a class of completely integrable equations that support a rich variety of dynamical regimes, from the classical KdV solitons to particles on a lattice interacting through long range forces.

4:20–5:00 The Forced Van der Pol Equation Revisited

John Guckenheimer
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The forced van der Pol equation is one of the classical examples in dynamical system theory. Nonetheless, the bifurcations of this system have not been thoroughly studied. This lecture will give describe new results of Kathleen Hoffman, Warren Weckesser and myself that are based upon investigation of the slow flow associated with the van der Pol system. We produce a global bifurcation diagram for the slow flow that encompasses the solutions of the system without canards.
