

**First Midterm Examination,
MAT 108, Differential Equations.**

Last Name

First Name

Student ID Number

Remarks:

- The test contains 5 pages, including the cover sheet.
- Do *not de-staple* the test.
- The use of books, notes, and calculators is not allowed.
- You can use scratch paper during the exam. However, you *cannot submit any work on your scratch paper* with the exam.
- Use both sides of the sheets.

Exercise I. Solve the following differential equations.

- $\frac{dy}{dt} = \frac{ty^2}{1+t^2}$.
- $\frac{dy}{dx} = \frac{-2x+5y}{2x+y}$ (Hint: first, show that it is homogeneous).
- $\frac{dy}{dt} = 1+t^2+y^2+t^2y^2$ (Hint: collect the y or the t terms).

Exercise II. Consider the following differential equation

$$\frac{dy}{dx} = -\frac{3xy + y^2}{x^2 + xy} \quad (1)$$

- (a) Show that (1) is not an exact differential equation.
- (b) Find the integrating factor that transforms (1) into an exact differential equation.
- (c) Find the solution to (1).

Exercise III. Consider the differential equation

$$\frac{dy}{dt} = -y(y+1)(y-1)(y-2) \quad (2)$$

- (a) Find the solution of (2).
- (b) Find and classify the equilibria of (2).

Exercise IV. Consider the following differential equation

$$\frac{dy}{dx} = P(x) + Q(x)y + R(x)y^2 \quad (3)$$

called the Riccati equation. Assume that y_1 is a particular solution.

- (a) Show that the change of variables $z = \frac{1}{y-y_1}$ transforms the differential equation (3) into a first order linear differential equation in the z variable.
- (b) Solve the differential equation $\frac{dy}{dx} = -2 - y + y^2$ knowing that $y = 2$ is a solution.