The solid angle form.

Here is an *extremely* useful formula which follows directly from the definition of exterior multiplication:

$$< \omega, v > \psi = \omega \wedge (\psi \llcorner v) + (\omega \wedge \psi) \llcorner v, \quad \omega \in V^*, \; v \in V, \; \psi \in \bigwedge^p V.$$

Suppose $p \in \mathbf{R}$. We define

$$\Omega_p \in \mathcal{A}^{n-1}(\mathbf{R}^n \sim \{0\})$$

be setting

$$\Omega_p(x) = |x|^p (\mathbf{e}^1 \wedge \cdots \wedge \mathbf{e}^n) \llcorner x, \quad x \in \mathbf{R}^n \sim \{0\}.$$

Exercise. Calculate $d\Omega_p$. If you do it correctly you will find that $d\Omega_p = 0$ if and only if $p = -n$. One calls Ω_{-n} the **solid angle form (on $\mathbf{R}^n$)**.

Exercise. Show that

$$\int_{\mathbf{S}^{n-1}} \Omega_{-n}$$

equals the $n-1$-area of $\mathbf{S}^{n-1}$.